This question paper contains 4+2 printed pages]

Your Roll No. ....

6741

B.A./B.Sc. (Hons.)/III **D**

MATHEMATICS—Unit XI

(Differential Equations—II)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *four* questions in all,

selecting *one* question from each Section.

All questions carry equal marks.

Section I

1. (*a*) Explain Charpit's method to solve a first order non-linear partial differential equation. 5

(*b*) Find the general solution of the partial differential equation : 4½

$$(x^2 - yz)\,p + (y^2 - xz)\,q = z^2 - xy.$$

2. (*a*) Find the complete integral of the equation : 4½

$$p^2 + q^2 = (x^2 + y^2)z.$$

(*b*) What is compatibility condition for two non-linear partial differential equations of first order ? Show that the equations :

$$xp = yq,\ z(xp + yq) = 2xy$$

are compatible and solve them. 5

Section II

3. (*a*) Reduce the equation :

$$y^2 \frac{\partial^2}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$$

to canonical form. 5

(*b*) Show that for the equation : 4½

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{2}{x+y}\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) = 0,$$

the Green's function is :

$$w(x, y, \xi, \eta) = \frac{(x+y)\left[2xy + (\xi - \eta)(x - y) + 2\xi\eta\right]}{(\xi + \eta)^3}.$$

4. (*a*) Reduce the equation :

$$x^2 r - y^2 t + pz - qy = x^2$$

to canonical form. 5

(*b*) Determine the solution of the equation :

$$\frac{\partial^2 z}{\partial x \partial y} + a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} + cz = f(x, y)$$

by Riemann-Volterra method, when the values of z and $\frac{\partial z}{\partial x}$ are prescribed on a curve C in the xy-plane. 4½

Section III

5. (a) The faces $x = 0$, $x = a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slab is described by the equation :

$$\theta = f(x),\ 0 \leq x \leq a.$$

Determine the temperature at a subsequent time t. 4½

(b) Derive the solution of the two-dimensional harmonic equation : 5

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0$$

in the region $0 \leq r \leq a$, $0 \leq \theta \leq 2\pi$ stisfying the conditions :

(i) V remains finite as $r \to 0$

(ii) $V = \sum C_n \cos n\theta$ on $r = a$.

6. (a) Find the potential function $\psi(x, y, z)$ satisfying the equation : 5

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0$$

in the region $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ and also satisfying the conditions :

(i) $\psi = 0$ on $x = 0, x = a, y = 0, y = b, z = 0$

(ii) $\psi = f(x, y)$ on $z = c$, $0 \le x \le a$, $0 \le y \le b$.

(b) A gas is contained in a cubical box of side a. Show that if c is the velocity of sound in the gas, the periods of free oscillations are : 4½

$$\frac{2a}{c\sqrt{n_1^2 + n_2^2 + n_3^2}},$$

where n, n_2, n_3 are integers.

Section IV

7. (*a*) Explain Monge's method of solving a p.d.e. of the form : 4½

$$Rr + Ss + Tt = V$$

where R, S, T and V are function of x, y, z, p and q.

(*b*) Solve the equations : 5

(*i*) $(3DD' - 2D'^2 - D^2)\, z = \sin(2x + 3y)$

(*ii*) $(D - 1)(D - D' + 1)\, z = 1 + e^y + \cos(x + 2y)$.

8. (*a*) Solve Monge's method : 4½

$$y^2 r - 2ys + t = p + 6y.$$

(*b*) Solve the equations : 5

(*i*) $(D - 3D')^2 (D + 3D')\, z = 5e^{3x+y}$

(*ii*) $(x^2 D^2 - y^2 D'^2)\, z = xy$.