This question paper contains 4+2 printed pages]

Your Roll No. .....

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B.A./B.Sc. (Honours)/III D

MATHEMATICS—Paper XVII and XVIII (*iv*)

(Integral Transforms and Boundary Value Problems)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

Symbols used have their usual meaning.

1. (*a*) Evaluate : 4.5

(*i*) $L(t \cos 2t)$

(*ii*) $L^{-1}\left(\frac{1}{s^2 - 5s + 6}\right)$.

P.T.O.

(*b*) If 4.5

$$L(F(t)) = f(s)$$

show that :

$$L\left(\int_0^t F(x)\,dx\right) = \frac{f(s)}{s}.$$

(*c*) By using Laplace transform technique, solve the initial value problem : 4.5

$$y''(x) + 2y'(x) + 5y(x) = e^{-x}\sin x,$$

$$y(0) = 0,\ y'(0) = 1.$$

2. (*a*) Find the basic Fourier series corresponding to the function : 5

$$f(x) = e^x,\ -\pi < x < \pi.$$

(*b*) A string is stretched between two fixed points $x = 0$ and $x = 1$ on the x-axis and released from rest from the position $y = \sin \pi x$. Find an expression for the subsequent displacement $y = y(x, t)$ at time t. 5

(*c*) Solve the following boundary value problem for $y(x, t)$: 5

$$y_{tt}(x, t) = a^2 y_{xx}(x, t),\ 0 < x < \pi,\ t > 0,$$

$$y(0, t) = 0,\ y(\pi, t) = 0$$

$$y(x, 0) = 0,\ y_t(x, 0) = 2\sin x.$$

3. (*a*) A slab $0 \leq x \leq 1$, is subjected to surface heat transfer, according to Newton's law of cooling, at its faces $x = 0$ and $x = 1$. The surface conductance H is the same on each face. If the medium $x < 0$ has temperature zero and the medium $x > 1$ has the subsequent temperature T, set up and solve the boundary value problem for steady-state temperature in the slab. 5

(*b*) Use the method of separation of variables to solve the following boundary value problem for the temperature

$u(x, t)$ in an infinite slab of material bounded by the planes $x = 0$ and $x = 1$: 5

$$u_t(x, t) = K\, u_{xx}(x, t),\ 0 \le x \le 1,\ t > 0$$

$$u_x(0, t) = 0,\ u_x(1, t) = 0$$

$$u(x, 0) = x$$

where K is a constant.

(c) Solve the following boundary value problem for the temperature $u(x, t)$ in an infinite slab of material bounded by the planes $x = 0$ and $x = \pi$: 5

$$u_t(x, t) = K\, u_{xx}(x, t),\ 0 \le x \le \pi,\ t > 0,$$

$$u(0, t) = 0,\ u(\pi, t) = 0,$$

$$u(x, 0) = \sin x,$$

where K is a constant.

4. (*a*) Express the function : 4.5

$$f(x) = \begin{cases} 1\ , & -1 < x < 1 \\ 1.5, & x = \pm 1 \\ 0\ , & x < -1,\ x > 1 \end{cases},$$

as a Fourier integral. Hence evaluate :

$$\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda}\, d\lambda.$$

(*b*) Using the Fourier integral theorem, show that if 4.5

$$f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0\ , & x < 0,\ x > \pi \end{cases}$$

then

$$f(x) = \frac{1}{\pi} \int_0^\infty \frac{\cos\ ax + \cos(\alpha(\pi - x))}{1 - \alpha^2}\, d\alpha,$$

$$-\infty < x < \infty.$$

Hence show that :

$$\int_0^\infty \frac{\cos\left(\frac{a\pi}{2}\right)}{1-a^2}\,d\alpha = \frac{\pi}{2}.$$

(*c*) Find the Fourier sine transform of the function : 4.5

$$f(x) = \frac{e^{-ax}}{x},\ a > 0.$$