

[This question paper contains 2 printed pages.]

6748

Your Roll No. :

B.A. / B.Sc. (Hons.) / III

D

MATHEMATICS – Paper XVII & XVIII (ii)

Boolean Algebra

Time : 2 Hours

Maximum Marks : 38

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*Attempt any two parts from each question.
First two questions carry 10(5+5) marks
each and last two 9(4.5+4.5) each.*

1. (a) Define a lattice. Show that product of two lattices is a lattice.
(b) Define a relatively complemented lattice. Prove that homomorphic image of a relatively complemented lattice is relatively complemented.
(c) Prove that the set of all principal ideals of a lattice L forms a lattice under \subseteq and is a sublattice of the lattice of all ideals of L .
2. (a) Show that any non modular lattice contains a sublattice isomorphic with the pentagonal lattice.
(b) Prove that a lattice is distributive if and only if $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$ for all $a, b, c. \in L$.

P.T.O.

- (c) If a and b are the elements of a modular lattice L then show that

$$[a \wedge b, a] \cong [b, a \vee b].$$

3. (a) If L is a Boolean algebra and $a \in L$, then show that $L \cong [0, a] \times [0, a']$.
- (b) Show that every finite Boolean algebra is isomorphic to a set with 2^n elements for some $n = 0, 1, 2, \dots$
- (c) Show that cross product of two distributive lattices is distributive.
4. (a) Find CN form of the function $f = (x \wedge (y' \vee z)) \vee z'$. Hence or otherwise find DN form of f .
- (b) Find the Boolean expression (in CN form) that defines the function f given by

x	y	z	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

- (c) Simplify the following function and draw the circuit.
 $bc + ab'cd + cd' + ac' + a'b'c' + b'c'd'$

(1000)