[This question paper contains 12 printed pages.]

5515 *Your Roll No.* ....

B.A. Prog. / III **D**

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt **all** *questions selecting* **two** *parts from each question.* **Unit I** *and* **Unit II** *are compulsory. In* **Unit III** *choose any of the options and attempt* **two** *questions from the same.*

UNIT - I
(Real Analysis)

1. (a) Define an open set. Which of the following sets are open ? Give an argument in support of your answer.

   (i) The set $\mathbb{Z}$ of integers.

   (ii) The set $\mathbb{Q}$ of rational numbers. (6)

   (b) Define limit point of a set. Prove that a finite set has no limit point. (6)

   (c) Prove that a function continuous on a closed interval [a,b] is bounded in [a,b]. (6)

2. (a) (i) Prove that every Cauchy sequence is a convergent sequence.

   (ii) Prove that $\lim_{n\to\infty} r^n = 0$, if $|r| < 1$. (7)

   (b) State and prove D'Alembert's Ratio Test for infinite series. (7)

   (c) Test the convergence of the following infinite series :

   (i) $\frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \frac{1}{\log 5} + \ldots\ldots\ldots\ldots\ldots\ldots$

(ii) $\sum \frac{1.3.5....(2n-1)}{2.4.6...2n} \frac{x^{2n+1}}{2n+1}, \qquad x > 0$ (7)

3. (a) State Weierstrass's M-Test of uniform convergence for a series of functions. Test for uniform convergence of the series of functions

$\sum f_n$, where $f_n(x) = \dfrac{x}{n(1+nx^2)}, x \in \mathbb{R}$. (6)

(b) Test for the convergence of the improper integrals :

(i) $\int_0^{\pi} \frac{\sqrt{x}}{\sin x} dx$

(ii) $\int_a^{\infty} \frac{1}{x^n} dx, \quad a > 0$ (6)

(c) Prove that

$$\int_0^1 \sqrt{1 - x^4}\, dx = \frac{1}{12}\sqrt{\frac{2}{\pi}}\left[\Gamma\left(\frac{1}{4}\right)\right]^2$$ (6)

UNIT II
(Computer Programming)

4. (a) (i) Define the control statements in C++ ? What are the different types of the control statements in C++ ? Are the control statements executable or non-executable ?

(ii) Whether the expressions given below are valid or invalid ? If invalid, give reasons :

```
float p, q, r, x ;
x * + r
p % q
log ( -3 ) + p / q
```

(6½)

(b) (i) What will be the output corresponding to the following program segment :

```
int n = 7 ;
cout << " ++n =" << ++n << ", n =" << n << "\n" ;
```

(ii) Give short notes on If statement in C ++.

Support your answer with a simple example for both the forms to explain the working of the If statement. (6½)

(c) Write a program to calculate and print the roots of a quadratic equation. (6½)

UNIT III (1)
(Numerical Analysis)

5. (a) Compare the Bisection method with Newton Raphson method for solving an equation.

Perform three iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$. (6)

(b) Find the Newton Raphson iterative Method, for finding $N^{1/3}$, where N is a real number and apply the method to N = 18, to obtain the result correct to four places of decimal, starting with the initial approximation as the left hand limit of the interval which contains the root. (6)

(c) Show that if a solution of system of linear equations converges in Gauss Jacobi sense, it always converges in Gauss Seidel sense. But converse may not be true. (6)

6. (a) Define an interpolating polynomial.

Using sin (0.1) = 0. 09983 and sin (0.2) = 0.19867, find an approximate value of sin (0.15), by lagrange's interpolation. (6)

Obtain a bound on the truncation error.

(b) If $f(x) = e^{ax}$, then show that

$\Delta^n f(x) = (e^{ah} - 1)^n e^{ax}$ (6)

(c) What is the basic requirement to the subintervals to apply Simpson's $\frac{1}{3}$rd rule ?

Apply Simpson's $\frac{1}{3}$rd rule (to the first seven points) & the trapezoidal rule (to the remaining) for the following data to get the value of the total integral :

x:	0.7	0.9	1.1	1.3	1.5	1.7	1.9	2.1
f(x):	0.64835	0.91360	1.16092	1.36178	1.49500	1.55007	1.52882	1.44513

What it will make the difference if the trapezoidal rule is applied to the left end rather than the right end ?

Which choice will be better & why ?

The exact value for the integral over the entire range is given as 1.81759. (6)

UNIT III (2)
(Discrete Mathematics)

5. (a) Define the following :

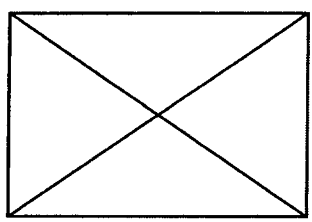
(i) The incoming degree and outgoing degree of a vertex in a directed graph.

(ii) A path in a graph.

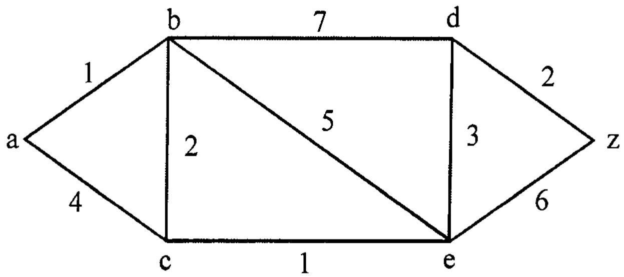
(iii) A directed multigraph.

(iv) A connected graph. (6)

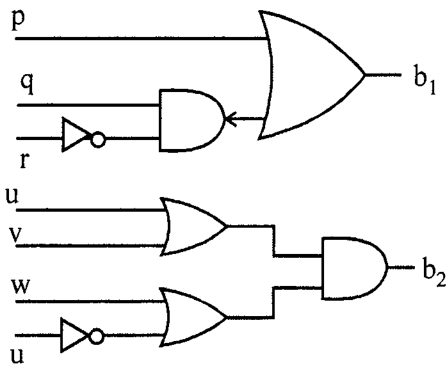
(b) Is the following graph planar ? If yes, draw the planar graph. (6)



(c) Obtain a shortest path from the vertex a to vertex z in the weighted graph. (6)



6. (a) Write Boolean expressions b_1 and b_2 represented by the following electronic circuit : (6)



(b) Write Boolean function corresponding to the Boolean expressions :

(i) $(\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \vee (\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \vee (x_1 \wedge x_2 \wedge x_3)$

(ii) $(x_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \vee (\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) \vee (x_1 \wedge x_2 \wedge \bar{x}_3)$

in tabular form specifying values of f corresponding to various possible combinations of values x_1, x_2 and x_3. (6)

(c) Show that the following statement is a tautology :

$(A \to B) \to [A \to (A \land B)]$ (6)

UNIT III (3)
(Mathematical Statistics)

5. (a) Calculate the mean deviation from the mean and the standard deviation of the series a, a + d, a + 2d, a + 3d, ... a + 2nd and prove that the later is greater than the former. (6)

(b) Cards are drawn one by one from a well shuffled pack until an ace appears. What is the probability that exactly n cards are dealt before the first ace appears. (6)

(c) If Z = aX + bY and r is the correlation coefficient between X and Y, then show that

$\sigma_Z^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2abr\sigma_X\sigma_Y$

and hence deduce that $r = \dfrac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{2\sigma_X\sigma_Y}$

symbols having usual meaning. (6)

6. (a) Let the random variable X have a Poisson distribution with parameter λ. Find the moment generating function of X and hence find mean, variance of X. (6)

(b) For a normal distribution with mean μ and standard deviation σ, prove that :

$$\mu_{2n} = 1.3.5 \ldots (2n-1)\,\sigma^{2n}$$

where μ_{2n} is the even order moments about the mean. (6)

(c) For two variables X and Y, the two regression lines are

$8X-10Y+66=0$ and $40X-18Y=214$. If $Var(X)=9$

Calculate (i) the mean values of X and Y

(ii) the correlation coefficient between X and Y

(iii) the standard deviation of Y. (6)

UNIT III (4)
(Mechanics)

5. (a) R is the resultant of two concurrent forces P and Q inclined at an angle; if P is doubled, then the resultant is doubled. Show that

$$\alpha = \sin^{-1}\left(\frac{16P^2 - 9Q^2}{16P^2}\right)^{1/2} \quad (6)$$

(b) A particle of weight W rest on a rough horizontal plane. If the angle of friction be μ, prove that the least force which will just make it move along the plane is

$$P = W\sin\mu.$$ (6)

(c) Find the centre of gravity of a quadrant of a circular disc of radius a. (6)

6. (a) A particle is moving with S.H.M of amplitude a and periodic time T. Prove that

$$\int_0^T \upsilon^2 dt = \frac{2\pi^2 a^2}{T}.$$ (6)

(b) Derive the expression $\frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2}$ for motion of a particle describing central orbit under an attraction 'p' per unit mass where $= \frac{1}{r}$. (6)

(c) If R be the horizontal range of the projectile and h its greatest height, prove that the initial speed is

$$\left[2g\left(h + \frac{R^2}{16h}\right)\right]^{1/2}.$$ (6)

UNIT III (5)

(Theory of Games)

5. (a) Solve graphically the LPP :

Minimize : $Z = 20x_1 + 10x_2$

subject to :

$$x_1 + 2x_2 \le 40$$
$$3x_1 + x_2 \ge 30$$
$$4x_1 + 3x_2 \ge 60$$
$$x_1, x_2 \ge 0$$ (6)

(b) Use simplex method to solve following problem :

Maximize : $Z = 5x_1 + 3x_2$

subject to : $x_1 + x_2 \le 2$

$$5x_1 + 2x_2 \le 10$$
$$3x_1 + 8x_2 \le 12$$
$$x_1, x_2 \ge 0$$ (6)

(c) Verify that the dual of dual is primal for the following LPP :

Maximize : $Z = 2x_1 + 5x_2 + 6x_3$

subject to :

$$5x_1 + 6x_2 - x_3 \le 3$$
$$-2x_1 + x_2 + 4x_3 \le 4$$
$$x_1, x_2, x_3 \ge 0$$ (6)

6. (a) Solve graphically the game whose pay-off matrix is

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \\ 2 & 2 \end{bmatrix}$$ (6)

(b) Find the range of values of p & q which will render (2,2) a saddle point of the game :

$$\begin{bmatrix} 4 & p & 2 \\ q & 5 & 7 \\ 10 & 3 & 9 \end{bmatrix}$$ (6)

(c) Reduce the following game to an LPP and hence solve :

$$\begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix}$$ (6)