[This question paper contains 12 printed pages.]

5514 *Your Roll No.* ....

B.A. Prog. / III **D**

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt **all** *questions selecting* **two** *parts from each question.* **Unit I** *and* **Unit II** *are compulsory. In* **Unit III** *choose any of the options and attempt* **two** *questions from the same.*

P.T.O.

UNIT - I

(Real Analysis)

1. (a) Define a closed set: Prove that the union of a finite number of closed sets is a closed set. What happens if the collection consists of infinite numbers of closed sets ? Justify with an example. (6)

   (b) Define limit point of a set. Show that the set $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ has zero as its only limit point. (6)

   (c) Define a uniform continuous function on an interval. Show that the function f(x) defined by $f(x) = \sin\frac{1}{x}$ is not uniformly continuous on $]0, \infty[$. (6)

2. (a) Define a monotonically increasing sequence and prove that a monotonically increasing sequence that is bounded above converges. (7)

   (b) Test the convergence of any **two** of the following infinite series :

   (i) $\sum \sqrt{n^4 + 1} - \sqrt{n^4}$

   (ii) $\sum \frac{\sqrt{n}}{\sqrt{n^2 + 1}} x^n, \qquad (x > 0)$

(iii) $\sum \frac{3.6.9\ldots\ldots\ldots3n}{7.10.13\ldots\ldots\ldots(3n+4)} x^n, \quad (x > 0)$ (7)

(c) State Leibnitz test for alternating series. Test the convergence and absolute convergence of the series :

$$\frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \frac{1}{\log 5} + \ldots\ldots\ldots\ldots\ldots$$ (7)

3. (a) Let f be a monotonically increasing function defined on a closed and bounded interval [a, b]. Prove that f is Riemann integrable on [a, b]. (6)

(b) Test for the convergence of the improper integrals :

(i) $\int_0^{\pi/2} \frac{x^m}{(\sin x)^n} dx$

(ii) $\int_0^{\infty} \frac{x^{m-1}}{1+x} dx$ (6)

(c) Prove that

$$\sqrt{\pi}\Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma\left(m + \frac{1}{2}\right)$$

and hence deduce that

$$\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \sqrt{2}\pi$$ (6)

UNIT - II
(Computer Programming)

4. (a) (i) What is the difference of 'a' & "a" in C++ ?

(ii) How do the computer detects whether an identifier used in a program is a constant or a variable or an array ? Support your answer with example, for all three.

(iii) How do the break statement differs with the continue statement. Show the difference with one simple example. (6½)

(b) (i) What will be the value of b if a = 10 initially when the following statement is executed ?
1. b = ++a + ++a & 2. b = ++a + a++

(ii) Give short notes on for structure in C++ with example. (6½)

(c) Write a program to calculate the factorial of an integer using while structure. (6½)

UNIT - III (1)
(Numerical Analysis)
(Use of Scientific calculator is allowed)

5. (a) Consider the equation

$$f(x) = x^4 - 3x^2 + x - 10 = 0$$

(i) Find the interval of unit length which contains smallest positive root of the equation.

(ii) Perform two iterations by the Bisection Method, taking the initial interval as considered in part (1).

(iii) Taking the midpoint of the last interval of part (2) as the initial approximation, obtain the root correct to two places of decimal, by the Newton Raphson Method. (6)

(b) Solve the system

$$\begin{pmatrix} 3 & 2 & 100 \\ -1 & 3 & 100 \\ 1 & 2 & -1 \end{pmatrix} x = \begin{pmatrix} 105 \\ 102 \\ 2 \end{pmatrix}$$

by Gauss Elimination Method, by applying scaling and carrying only two places of decimal. (6)

(c) Find the inverse of the coefficient matrix of the system

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$$

by Gauss Jordan method with partial pivoting & hence solve the system. (6)

6. (a) Define an interpolating polynomial.

Using sin (0.1) = 0.09983 and sin (0.2) = 0.19867, find an approximate value of sin (0.15), by Lagrange's interpolation.

Obtain a bound on the truncation error. (6)

(b) Evaluate $\int_0^1 \frac{1}{1+x} dx$, correct to three decimal places by Trapezoidal rule and Simpson's $1/3^{rd}$ rule with h = 0.25.

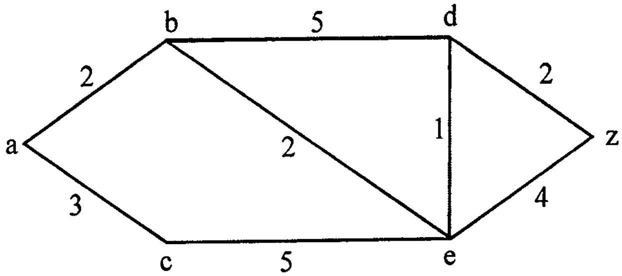
Also determine which method yields more accurate result & why ? (6)

(c) What is the basic requirement for the integral in Gauss Legendre's Quadrature formula ?

Evaluate the integral $\int_0^1 x\,dx$, using Gauss Legendre's 3 - point formula. Also, compute the error. (6)

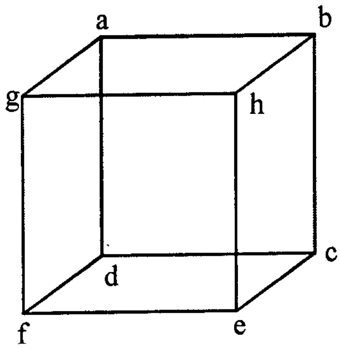
UNIT – III (2)
(Discrete Mathematics)

5. (a) Find the length of a shortest path between a and z in the given weighted graph (6)



(b) For any planar connected graph, prove that $n - e + r = 2$ where n, e and r are the number of vertices, edges and regions of the graph respectively. (6)

(c) Find the Hamiltonian circuit in the given graph.



(6)

6. (a) Let :

$$E(x_1, x_2, x_3) = \overline{\overline{(x_1 \vee x_2)} \vee (\overline{x_1} \wedge x_3)}$$

be a Boolean expression over the two valued Boolean algebra. Write $E(x_1, x_2, x_3)$ into conjunctive normal form. (6)

(b) Prove the equation :

$$(a \vee \bar{b} \vee d) \wedge (a \vee b \vee \bar{c}) \wedge (a \vee c \vee d) = a \vee (b \wedge d) \vee (\bar{c} \wedge d)$$

(6)

(c) Write the truth tables for $p \rightarrow q$ and $p \leftrightarrow q$ and show that :

$p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$. (6)

UNIT - III (3)
(Mathematical Statistics)

5. (a) Show that for any discrete distribution, standard deviation is not less than mean deviation from mean. (6)

(b) The contents of urns I, II and III are as follows :

1 white, 2 red and 3 black balls.
2 white, 3 red and 1 black balls, and
3 white, 1 red and 2 black balls.

One urn is chosen at random and two balls drawn. They happen to be white and red, What is the probability that they come from urns I, II and III ? (6)

(c) If X_1, X_2, X_3 are uncorrelated variables each having the same standard deviation. Obtain the correlation coefficient between $X_1 + X_2$ and $X_2 + X_3$. (6)

6. (a) Prove that the recurrence relation for the binomial distribution is :

$$\mu_{r+1} = pq\left(nr\mu_{r-1} + \frac{d\mu_r}{dp}\right)$$

Where μ_r is the r^{th} moment about the mean ? Hence obtain μ_2, μ_3 and μ_4. (6)

(b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. Given that if

$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t \exp\left(-\frac{x^2}{2}\right) dx$, Then f(0.496) = 0.19 and f(1.405) = 0.42. (6)

(c) For two variables X and Y, the two regression lines are

$X + 2Y - 5 = 0$ and $2X + 3Y - 8 = 0$. If $Var(X) = 12$

Calculate (i) the mean values of X and Y

(ii) The correlation coefficient between X and Y

(iii) the standard deviation of Y. (6)

Unit – III (4)
(Mechanics)

5. (a) Three like parallel forces P,Q,R act the vertices of the triangle ABC. If their resultant passes through the circumcentre in all cases, whatever be the common direction of the forces, show that

$$\frac{P}{\sin 2A} = \frac{P}{\sin 2B} = \frac{P}{\sin 2C} \qquad (6)$$

(b) A particle of weight W rest on a rough horizontal plane. If the angle of friction be μ, prove that the least force which will just make it move along the plane is

$$P = W\sin\mu. \qquad (6)$$

(c) Find the centre of gravity of a quadrant of a circular disc of radius a. (6)

6. (a) A particle is moving with S.H.M of amplitude a and periodic time T. Prove that

$$\int_0^T v^2 dt = \frac{2\pi^2 a^2}{T}. \qquad (6)$$

(b) Derive the expression $\frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2}$ for motion of a particle describing central orbit under an attraction 'p' per unit mass where $u = \frac{1}{r}$. (6)

(c) A particle just goes over a wall height h meters and at a distance d meters apart from the point of projection, and later it hits a mark at a height 'h' meters and distance 2d meters. Show that the velocity of projection 'v' is given by

$$\frac{4v^2}{g} = \frac{4d^2 + 9h^2}{h}$$ (6)

Unit – III (5)
(Theory of Games)

5. (a) Solve graphically the following LPP

Maximize $Z = 10x_2 - 2x_1$

subject to : $x_1 - x_2 \geq 0$

$-x_1 + 5x_2 \geq 5$

$x_1, x_2 \geq 0$ (6)

(b) Use simplex method to solve following problem :

Maximize $Z = 4x_1 + 10x_2$

subject to : $2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

$x_1, x_2 \geq 0$ (6)

(c) Apply principle of duality to solve :

Minimize $Z = -2x_1 + 3x_2 + 4x_3$

subject to : $-2x_1 + x_2 \geq 3$

$-x_1 + 3x_2 + x_3 \geq -1$

$x_1, x_2, x_3 \geq 0$ (6)

6. (a) Explain the max-Min and Min-Max principle used in game theory. Determine the saddle point of a game whose pay-off matrix is :

Player B

		B_1	B_2
Player A	A_1	−1	6
	A_2	2	4
	A_3	−2	−6

(6)

(b) Solve graphically the rectangular game whose pay-off matrix is :

$$\begin{bmatrix} 19 & 15 & 7 & 6 \\ 0 & 20 & 15 & 5 \end{bmatrix}$$ (6)

(c) Transform the matrix game :

$$\begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 3 & 2 & -3 \end{bmatrix}$$

into its corresponding primal and dual LPP and solve. (6)