[This question paper contains 12 printed pages.]

338 *Your Roll No.* ....

B.A. Prog. / III **E**

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt all questions selecting two parts from each question. ***Unit I and Unit II are compulsory.***

In ***Unit III*** *Choose any of the options and attempt two questions from the same.*

P.T.O.

Unit —I

(Real Analysis)

1. (a) State Bolzano Weierstrass theorem for real numbers. Can any of the conditions of Bolzano Weierstrass theorem be relaxed ? Justify. (6)

   (b) Define the following terms with examples:

   (i) Bounded set

   (ii) limit point of set of real numbers (6)

   (c) Let f be a function defined on [0,1] by setting

   $f(0) = 0$, $f(x) = \frac{1}{2^n}$, when $\frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}$,

   $n = 0,1,2,3......$ Show that f is discontinuous at the points $x = \frac{1}{2^n}$, $n = 1,2,3......$ (6)

2. (a) Define a Cauchy sequence and prove that the sequence $< a_n >$ defined by:

   $a_n = 1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}$ Is not convergent. (7)

(b) Test the convergence of any two of the following infinite series:

(i) $\sum e^{-n^2}$

(ii) $\sum \frac{1}{\sqrt{n(n+1)}}$

(iii) $\sum 2^{-n-(-1)^n}$ (7)

(c) Define conditional convergence of a series of numbers. Show that the series $\sum (-1)^{n-1} \frac{1}{n}$ is conditionally convergent. (7)

3. (a) Show that the sequence $\{f_n\}$, where

$$f_n \frac{n}{x+n}$$

is uniformly convergent in [0, K] whatever k may be, but not uniformly convergent in [0, ∞]. (6)

(b) Test the convergence of the improper integral

$$\int_0^{\pi} \frac{\sqrt{x}}{\sin x}\, dx \quad (6)$$

(c) Find the radius of convergence and the exact interval of convergence of the following power series:

$$\sum \frac{(n+1)}{(n+2)(n+3)} x^n \quad (6)$$

Unit-II

(Computer Programming)

4. (a) Write a program to find roots of a quadratic equation. (6½)

(b) Write a short note on type declaration statement. What reserved words are used for declaring different types of data? (6½)

(c) Give the general form of the while loop. Explain, how it works, with an example. (6½)

Unit-III(1)

(Numerical Analysis)

5. (a) For the following system of equations:

$$\begin{pmatrix} -3 & 1 & 0 \\ 2 & -3 & 1 \\ 0 & 2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$$

starting with $x_1 = x_2 = x_3 = 0$, using Gauss-Seidel Method, find the solution after performing three iterations. (6)

(b) Describe bisection method to find a root of an equation: $f(x) = 0$ (6)

(c) Find the inverse of the coefficient matrix of the system

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$$

by the Gauss-Jordan method and hence solve the system. (6)

6. (a) Given p(0)=1, p(1)=3, p(3)=55. Find the Langrange quadratic interpolating polynomial, which fits the given data. Estimate p(1.5). (6)

(b) Evaluate:

$$\int_1^2 \frac{\sin x}{x}\,dx,$$

Using Simpson's $\frac{3}{8}$ rule with 6 sub-intervals. (6)

(c) Evaluate the integral $\int_0^1 \frac{dx}{1+x^2}$, using 3 point Gauss-Legendre's quadrature formula. (6)

Unit-III(2)

(Discrete Mathematics)

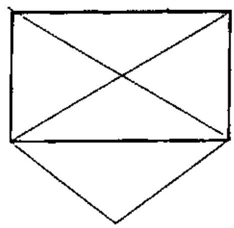
5. (a) Define the following:

(i) A directed graph;

(ii) The incoming degree and outgoing degree of a vertex in a directed graph;

(iii) A connected graph. (6)

(b) Find a Hamiltonian circuit in the graph:



(6)

(c) Show that any simple, connected graph with 31 edges and 12 vertices is not planar. (6)

6. (a) Let

$E(x_1,x_2,x_3) = (x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (x_2 \wedge x_3)$ be a Boolean expression over the two valued Boolean algebra. Write $E(x_1,x_2,x_3)$ in both conjunctive and disjunctive normal forms. (6)

(b) Simplify the circuit represented by

$$f = (a \wedge \bar{c} \wedge \bar{d}) \vee (a \wedge \bar{b} \wedge d) \vee (a \wedge c \wedge \bar{d}) \qquad (6)$$

(c) Write the truth tables to show that $p \leftrightarrow q = (p \to q) \wedge (q \to p)$ (6)

Unit-III(3)

(Mathematical Statistics)

5. (a) The first four moments of a distribution about the value 4 are -1.5, 17, -30, 108.

Find mean, β_1 and β_2. (6)

(b) Three urns A, B, C have the following coloured balls:

A: 6 red, 4 white; B: 2 red, 6 white ; C :1 red, 8 white. An urn is chosen at random; a ball drawn turns out to be red. Find the chance that urn A is chosen. (6)

(c) Show that the correlation coefficient r between two variables lies between -1 and 1. (6)

6. (a) For Binomial distribution, find β_1 and γ_1. (6)

(b) Show that the mean deviation from the mean of the normal distribution is about $\frac{4}{5}$ of its standard deviation. (6)

(c) Two random variables have the least squares regression lines $3x + 2y = 26$ and $6x + y = 31$. Find the mean values and the regression coefficients. (6)

Unit-III(4)

(Mechanics)

5. (a) Three forces each equal to 'P' act along the sides of a triangle ABC in order, prove that the resultant is : $P(1-8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2})^{1/2}$ and find the distance of its line of action from one angular point, and here it cuts one side of the triangle. (6)

(b) Find the centre of gravity of a sector of a circle in which the surface density varies as the Distance from the centre. (6)

(c) A light ladder is supported on a rough floor and lean against a smooth wall. How far up the ladder can a man climb without slipping taking place ? (6)

6. (a) A gun is mounted on a hill of height 'h' above a sea level plain. Show that, if the resistance of the air is neglected, the greatest horizontal range for given muzzle velocity 'v' is obtained by firing at an angle of elevation 'θ' such that:

$$\cos ec^2\theta = 2(1+\frac{gh}{v^2})$$ (6)

(b) The displacement of a moving point at any point is given by:

$x = a \cos kt + b \sin kt$

Show that the point executes a simple harmonic motion. (6)

(c) Establish the formula

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2}, \quad \theta = hu^2, \text{ where } u = \frac{1}{r}$$

for the motion of a particle describing a central orbit under an attraction F per unit mass. (6)

Unit-III(5)

(Theory of Games)

5. (a) Show that the following system of linear equation has a degenerate solution:

$2x_1 + x_2 - x_3 = 2$

$3x_1 + 2x_2 + x_3 = 3$ 6)

(b) Use two – phase simplex method to solve following problem :

$$\min imize\ z = 4x_1 + 2x_2$$

subject to the constraints

$3x_1 + x_2 \geq 27$

$x_1 + x_2 \geq 21$

$x_1, x_2 \geq 0$ (6)

(c) Find the dual of the following linear programme:

Minimize : $Z = x_1 + x_2 + 2x_3$

Subject to $x_1 + 2x_2 \geq 3$

$x_2 + 7x_3 \leq 6$

$x_1 - 3x_2 + 5x_3 = 5$

$x_1, x_2 \geq 0$, x_3 *unrestricted* (6)

6. (a) Use graphical method to solve the rectangular game whose pay off matrix is:

$$\begin{pmatrix} 1 & 0 & 4 & -1 \\ -1 & 1 & -2 & 5 \end{pmatrix}$$ (6)

(b) Use dominance to solve the following game:

$$\begin{pmatrix} 6 & 8 & 6 \\ 4 & 12 & 2 \end{pmatrix}$$ (6)

(c) Solve the following LPP using Big M method:

Mix *imize* $z = 12x_1 + 20x_2$

subject to the constraints :

$6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$

$x_1, x_2 \geq 0$ (6)