[This question paper contains 12 printed pages.]

337 *Your Roll No.* ....

B.A. Prog. / III **E**

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

(Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt all questions selecting two parts from each question. Unit I and Unit II are compulsory.

In Unit III Choose any of the options and attempt two questions from the same.

Unit – 1

(Real Analysis)

1. (a) Define neighbourhood of a point. Show that]0,1[is neighbourhood of each of its points. (6)

   (b) Define the following terms with examples:

      (i) Supremum of a set

      (ii) Closed set. (6)

   (c) Let a function f is continuous in [a,b] and $f(a) \neq f(b)$, then prove that f assumes every value between $f(a)$ and $f(b)$. (6)

2. (a) Define convergent sequence. Prove that if $< a_n >$, $< b_n >$, $< c_n >$ be three sequences such that $< a_n \leq b_n \leq c_n >$, $\forall n$ and $\lim a_n = \lim c_n = l$ then $\lim b_n = l$. (6)

   (b) Test the convergence of the following series:

      (i) $\sum \frac{n+1}{n^p}$

      (ii) $\sum \frac{n!}{n^n}$ (6)

(c) Define upper sum and lower sum of a bounded function f(x) defined on a closed interval [a,b] corresponding to a partition of [a,b]. Define Riemann integrability of f(x). Show that the

function $f(x)=\begin{cases}0 & \text{if } x \text{ is irrational}\\ 1 & \text{if } x \text{ is rational}\end{cases}$

is not Riemann integrable. (6)

3. (a) Test the convergence of the improper integrals:

(i) $\int_a^\infty \frac{dx}{x^n}$ (a > 0)

(ii) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x^n} dx$ (7)

(b) Show that the series $\sum_1^\infty \frac{x}{n(1+nx^2)}$ converges uniformly for all real x. (7)

(c) (i) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

(ii) Find the radius of convergence of the power series: $\sum \frac{x^n}{n!}$ (7)

Unit-II

(Computer Programming)

4. (a) Write a short note on the break statement. Explain its use with an example. (6½)

(b) Given a five digit integer, write a program to reverse the number and print it. (6½)

(c) Write a program to evaluate

$$f(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + (4 - x^2)^3$$

for x = 0.05, 0.55 (6½)

Unit III (1)

(Numerical Analysis)

5. (a) Find the root of the equation

$x^n = 100$, correct to 4 places of decimals, using Newton-Raphson method. (6)

(b) Solve the following system of equations, by using Gauss-Jordan elimination method :

$x_1 + x_2 + x_3 + x_4 = 2$

$x_1 + x_2 + 3x_3 - 2x_4 = -6$

$2x_1 + 3x_2 - x_3 + 2x_4 = 7$

$x_1 + 2x_2 + x_3 - x_4 = -2$ (6)

(c) Using the Gauss Elimination method, solve the system of equations:

$10x_1 - x_2 + 2x_3 = 4$

$x_1 - 10x_2 - x_3 = 3$

$2x_1 + 3x_2 + 20x_3 = 7$ (6)

6. (a) Evaluate :

$\int_0^1 \frac{dx}{\sqrt{x^3 + 1}}$, using Simpson's one-third rule with 10 sub-intervals. (6)

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(b) Let $f(x) = \log(1 + x)$, $x_0 = 1$, $x_1 = 1.1$. Use linear interpolation to calculate an approximate value of $f(1.04)$. (6)

(c) Evaluate the integral $\int_{-1}^{1} e^{-x^2} \cos x \, dx$, using Gauss-Legendre 3-point formula. (6)

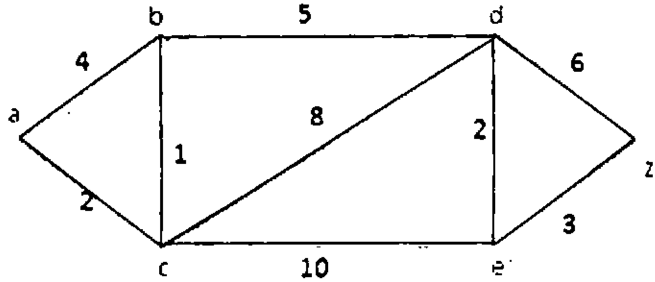
Unit-III(2)

(Discrete Mathematics)

5. (a) What is weighted graph? Define length of path in a weighted graph. (6)

(b) Show that in any simple, connected, planar graph, $e \leq 3v - 6$. Give an example. (6)

(c) Find the length of a shortest path between a and z in the given weighted graph (6)



6. (a) What is a combinatorial circuit ? Find the combinatorial circuit corresponding to the Boolean expression. $(a \wedge (\overline{b} \vee c)) \vee b$ and write the logic table for the circuit obtained. (6)

(b) Prove the equation:

$$(a \vee \overline{b} \vee d) \wedge (a \vee b \vee \overline{c}) \wedge (a \vee c \vee d)$$

$$= a \vee (b \wedge d) \vee (\overline{c} \wedge d)$$ (6)

(c) Define the following :

(i) an AND gate

(ii) an OR gate

(iii) a NOT gate (6)

Unit-III(3)

(Mathematical Statistics)

5. (a) The first three moments of a distribution about the value 2 of the variable are 1, 16, -40. Show that the mean is 3, variance is 15 and $\mu_3 = -86$. Also show that the first three moments about x = 0 are 3, 24, 76. (6)

(b) Show that $E(X_1X_2 \ldots\ldots X_n) = E(X_1)E(X_2) \ldots E(X_n)$ where $X_1, X_2 \ldots\ldots X_n$ are independent random variables. (6)

(c) Find the mean, variance, moment generating function of the Poisson distribution. (6)

6. (a) Fit a parabola to the following data taking x as the independent variable:

X :	1	2	3	4	5	6	7	8	9
Y :	2	6	7	8	10	11	11	10	9

(6)

(b) For the normal distribution, show

$\beta_1 = 0, \beta_2 = 3$ (6)

(c) If X, Y and Z are uncorrelated variables each having same standard deviation, find the correlation coefficient between X + Y and Y + Z. (6)

Unit-III (4)

(Mechanics)

5. (a) Two heavy particles of weight w and w' are connected by a light inextensible string and hang over a fixed smooth circular cylinder of radius 'a', the axis of which is horizontal if and θ and θ' are the inclinations to the vertical of the radii drawn to the particles, slow that :

$$\frac{\sin\theta}{\sin\theta'} = \frac{w}{w'}$$ (6)

(b) Find the mass centre of a wire bent into the form of an isosceles right angled triangle. (6)

(c) Show that any force system in the fundamental plane may be reduced either to a single force or to a couple. (6)

6. (a) A gun is fired from a moving platform, and the ranges of the shot are obsereved to the R and S, when the platform is moving backward and forward respectively, with velocity v. Prove that the elevation of the gun is :

$$\tan^{-1}\left[\frac{g(R-S)^2}{4v^2(R+S)}\right]$$ (6)

(b) A particle describe an elliptic orbit under a central force towards one focus S. If v_1 is the speed at the end B of the minor axis and v_2, v_3 the speeds at the ends A, A' of the major axis, show that :

$v_1^2 = v_2 v_3$ (6)

(c) If a and b are the velocities of a planet when it is respectively, nearest and farthest from the sun, prove that :

$$(1 - e)a = (1 + e)\ b$$

Where e is the eccentricity of the planets orbit. (6)

Unit-III(5)

(Theory of Games)

5. (a) Solve graphically the LPP:

Maximize $Z = 2x_1 - x_2$

subject to:

$x_1 + x_2 \leq 5$

$x_1 + x_2 \geq 8$

$x_1, x_2 \geq 0$ (6)

(b) Use simplex method to solve following problem:

Maximize : $Z = 6x_1 - 2x_2$

subject to; $2x_1 - x_2 \leq 2$

$x_1 \leq 4$

$x_1, x_2 \geq 0$ (6)

(c) Find the dual of the LPP:

$$\text{Maximize} : Z = x_1 - 3x_2 - 2x_3$$

$$\text{subject to} : 3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 - 4x_2 \geq 12$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

$x_1, x_2 \geq 0$, x_3 *is unrestricted.* (6)

6. (a) Use dominance to solve the following game:

$$\begin{pmatrix} -2 & 15 & -2 \\ -5 & -6 & -4 \\ -5 & -20 & -8 \end{pmatrix}$$ (6)

(b) Transform the matrix game

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

into its corresponding primal and dual linear programming problem and solve. (6)

(c) Use two-phase simplex method to maximize

$z = 2x_1 + x_2 - x_3$

subject to the constraints:

$4x_1 + 6x_2 + 3x_3 \leq 8$

$3x_1 - 6x_2 - 4x_3 \leq 1$

$2x_1 + 3x_2 - 5x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$ (6)