[This question paper contains 12 printed pages.]

1405 *Your Roll No.* ....

B.A. (Programme)/III **E-I**

MATHEMATICS–Paper III

(Selected Topics in Mathematics)

(NC–Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt six questions in all selecting two parts from each question.

Unit I and Unit II are compulsory and contain four questions.

In Unit III choose any of the options and attempt two questions from the same.

Marks are indicated against each question.

Unit -I
(Real Analysis)

1. (a) Define neighbourhood of a point. Show that]0,1[is neighbourhood of each of its points. Also show that the set Z of integers is not a neighbourhood of any of its points. (7)

   (b) Define limit point of a set. State Bolzano Weierstrass Theorem and show by an example that condition of boundedness in the theorem cannot be relaxed. (7)

   (c) Show that the function f defined on R by

$$f(x)=\begin{cases}1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational}\end{cases}$$

   is discontinuous at every point. (7)

2. (a) Define a monotonically increasing sequence and prove that a monotonically increasing sequence that is bounded above converges. (9)

   (b) Test the convergence of any two of the following series:

   (i) $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+\frac{1.2.3.4}{3.5.7.9}+.......$

(ii) $\sum 2^{-n-(-1)^n}$

(iii) $\frac{1}{3.7}+\frac{1}{4.9}+\frac{1}{5.11}+\frac{1}{6.13}+\ldots\ldots\ldots\ldots$ (9)

(c) State Leibnitz Test for the convergence of an alternating series. Test the convergence and the absolute convergence of the series:

$$\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\ldots\ldots\ldots$$ (9)

3. (a) Let f be a continuous function defined on a closed and bounded interval $[a,b]$. Prove that f is Riemann integrable on $[a,b]$. (7)

(b) Show that the integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

converges if and only if $m > 0$, $n > 0$. (7)

(c) Define Beta function. Prove that:

(i) $\Gamma\left(\frac{1}{2}\right)=1$

(ii) $\beta(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx, m>0, n>0$. (7)

Unit-II
(Computer Programming)

4. (a) Write a program to check whether a square matrix is symmetric or not. (9)

   (b) (i) Write a short note on the continue statement. Explain with example, how it is used?

   (ii) Write functions to add and subtract two complex numbers *(a + ib)* and (c + *id*). (9)

   (c) Given a five digit integer. Write a program to reverse the number and print it. (9)

Unit III(4)
(Numerical Analysis)
(Use of scientific calculator is allowed)

5. (a) Find the root of the equation

   $e^x = 2x + 1$

   correct to 4 decimal places, using Newton-Raphson method. (9)

   (b) Use the Gauss- Jacobi iteration method to solve the system of equations:

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ 1 \end{pmatrix}$$

Perfom two iterations only. (9)

(c) Find the inverse of the coefficient matrix of the system

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$$

by Gauss Jordan method and hence solve the system. (9)

6. (a) Find the unique interpolating polynomial of degree 2 or less ,which fits the given data:

p(0) = 1 , P (1) = 27, p(4) = 64 by using Lagrange's method.

Hence evaluate p (3). (9)

(b) Evaluate the integral $\int_{-1}^{1} x^2 e^{-x} dx$, by Simpson's 1/3rd rule with h = 0.25. (9)

(c) Evaluate the integral $\int_0^1 \frac{\sin x}{x}\, dx$, with a four-term Gaussian formula. (9)

Unit-III(2)
(Discrete Mathematics)

5. (a) Define the following:

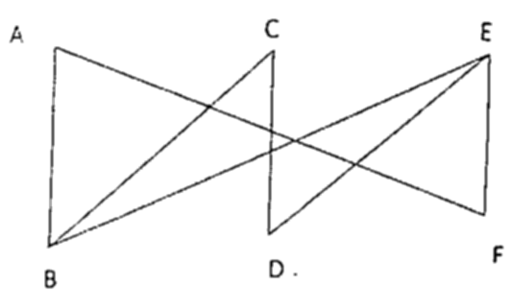
   (i) The directed multigraph;

   (ii) A path in a graph;

   (iii) A connected graph. (9)

   (b) Prove that a connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has even degree. (9)

   (c) Determine whether the given graph is planar. If so, draw it so that no edges cross.



(9)

6. (a) Let:

$$E(x_1, x_2, x_3) = \overline{\overline{(x_1 \vee x_2)} \vee (\overline{x_1} \wedge x_3)}$$

be a Boolean expression over the two valued Boolean algebra. Write $E\ (x_1,\ x_2,\ x_3)$ into conjunctive normal form. (9)

(b) Prove the equation:

$$(a \vee \bar{b} \vee d) \wedge (a \vee b \vee \bar{c}) \wedge (a \vee c \vee d) = a \vee (b \wedge d) \vee (\bar{c} \wedge d)$$

(9)

(c) Show that the following statement is a tautology:

$(A \rightarrow B) \rightarrow [(\bar{A} \rightarrow B) \rightarrow B]$

Where A denotes the negation of A. (9)

Unit III(3)

(Mathematical Statistics)

5. (a) Define the moment generating function. Show that the sum of two independent random variables is equal to the product of their moment generating functions. (9)

(b) A and B are two weak students of statistics and their chances of solving a problem in statistics correctly are $\frac{1}{6}$ *and* $\frac{1}{8}$ respectively, If the probability of their making a common error is $\frac{1}{525}$ and they obtain the same answer, find the probability that their answer is correct. (9)

(c) Let $U = aX + bY$ and $V = bX - aY$, where X and Y are measured from their means. If the correlation coefficient between X and Y is p and U and V are uncorrelated, prove that

$$ab(\sigma_X^2 - \sigma_Y^2) = \rho\sigma_X\sigma_Y(a^2 - b^2)$$

symbols having usual meaning. (9)

6. (a) Find the probability that almost 5 defective fuses will be found in a box of 200 fuses, if experience show that 2% of such fuses are defective. Given $e^{-4} = 0.0183$. (9)

(b) For a normal distribution with mean μ and standard deviation σ, prove that:

$$\mu_{2n} = 1.3.5 \ldots (2n - 1)\sigma^{2n}$$

where μ_{2n} is the even order moments about the mean. (9)

(c) Obtain the equation of the line of regression of y on x for the following data:

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Also obtain an estimate of y for x= 6.2. (9)

Unit III(4)

(Mechanics)

5. (a) Two heavy particles of weights w, w' are connected by a light inextensible string and hang over a fixed smooth circular cylinder of radius 'a'. the axis of which is horizontal.

if θ *and* θ' are the inclination to the vertical of the radii drawn to the particles, show that:

$$\frac{\sin\theta}{\sin\theta'}=\frac{w}{w'}.$$ (9)

(b) Two weights w_1 and w_2 rest on a rough plane inclined at an angle α to the horizontal and are connected by a string which lies along the line of greatest slope. If μ_1 and μ_2 are their coefficients of friction with the plane, and $\mu_1 > \tan\alpha > \mu_2$, prove that, if they are both on the point of slipping,

$$\tan\alpha = \frac{\mu_1 w_1 + \mu_2 w_2}{w_1 + w_2} \quad (9)$$

(c) Find the mass centre of a wire bent into the form of an isosceles right-angled triangle. (9)

6. (a) A particle is performing a S.H.M of period T about a centre 'O' and it passes through a point P, where OP=b with velocity v in the direction OP. Prove that the time which elapses before its return to P is

$$\left(\frac{T}{\pi}\right)\tan^{-1}\left(\frac{vT}{2\pi b}\right). \quad (9)$$

(b) Derive the expression $\frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2}$ for motion of a particle describing central orbit under an attraction 'p' per unit mass where = $\frac{1}{r^2}\theta = hu^2$. (9)

(c) If v_1 *and* v_2 be the velocities at the ends of a focal chord of a projectile's path and 'u', the horizontal component of velocity, show that

$$\frac{1}{v_1^2}+\frac{1}{v_2^2}=\frac{1}{u^2}$$ (9)

Unit-III (5)

(Theory of Games)

5. (a) Solve graphically the following LPP

   Maximize $Z = 10x_2 - 2x_1$

   subject to: $x_1 - x_2 \geq 0$

   $-x_1 + 5x_2 \geq 5$

   $x_1, x_2, \geq 0$ (9)

   (b) Use simplex method to solve following problem:

   Maximize: $Z = 4x_1 + 5x_2$

   subject to: $x_1 + x_2 \geq 1$, $-2x_1 + x_2 \leq 1$

   $4x_1 - x_2 \geq 1$

   $x_1, x_2 \geq 0$ (9)

   (c) Using principle of duality solve:

   Maximize $Z = 3x_1 - 2x_2$, subject to

   $x_1 + x_2 \leq 5$

   $-x_2 \leq -1$

$0 \leq x_1 \leq 4$

$0 \leq x_2 \leq 6$ (9)

6. (a) Explain the max-Min and Min-Max principle used in game theory. Determine the saddle point of a game whose pay-off matrix is:

$$\begin{array}{ccc} & & \text{Player } B \\ & & \begin{array}{cc} B_1 & B_2 \end{array} \\ \text{Player } A & \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} -1 & 6 \\ 2 & 4 \\ -2 & -6 \end{bmatrix}. \end{array}$$ (9)

(b) Solve graphically the rectangular game whose pay-off matrix is:

$$\begin{bmatrix} 0 & 4 & -8 & -5 & 1 \\ 1 & 5 & 8 & -4 & 0 \end{bmatrix}$$ (9)

(c) Transform the matrix game:

$$\begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix}$$

into its corresponding primal and dual LPP and solve. (9)