[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5274 **D** **Your Roll No...............**

Unique Paper Code : 235551

Name of the Course : **B.A. Programme**

Name of the Paper : Analysis

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **3** sections.
3. Each section carries **25** marks.
4. Attempt any **two** parts from each question in each section.
5. Marks are indicated against each question.

SECTION I

1. (a) Prove that the union of a finite collection of closed sets in R is closed. Give an example to show that the result may not be true for an infinite collection of closed sets. (6.5)

(b) Let $S = \left\{\frac{1}{n} : n \varepsilon N\right\}$. Prove that S has only one limit point, namely 0. (6.5)

(c) Let f be a continuous function defined on [a,b] such that f(a) and f(b) are of opposite signs. Then show that, there exists some point c ∈ (a,b) such that f(c) = 0. (6.5)

P.T.O.

2. (a) Define a closed set. Prove that every closed interval is a closed set. (6)

(b) Let f be a function on R defined by

$$f(x) = 1, \text{ when } x \text{ is rational and,}$$

$$f(x) = -1, \text{ when } x \text{ is irrational}$$

Prove that f is discontinuous at every point of R. (6)

(c) Define a uniformly continuous function. Show that $f(x) = x^2, x \in R$ is uniformly continuous on $(-2,2)$. (6)

SECTION II

3. (a) Define convergent sequence. Show that limit of a sequence, if exists, is unique. (6.5)

(b) Prove that the sequence $\langle a_n \rangle$ defined by the relation

$$a_1 = 1, a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots\dots\dots + \frac{1}{(n-1)!}, \ n \geq 2$$

converges. (6.5)

(c) State Cauchy's Convergence Criterion for sequences. Apply it to show that the sequence $\langle S_n \rangle$ defined by $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots\dots\dots + \frac{1}{n}, \ n \geq 1$ does not converge. (6.5)

4. (a) State and prove the Cauchy's General Principle for convergence of an infinite series $\sum_{n=1}^{\infty} u_n$. (6)

(b) Test for convergence any **two** of the following series :

(i) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+..........$

(ii) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$

(iii) $\sum_{n=1}^{\infty}e^{-n^2}$ (6)

(c) Define absolute and conditional convergence of an alternating series. Show that the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...........$ is conditionally convergent. (6)

SECTION III

5. (a) Prove that a bounded function f is integrable on a bounded closed interval [a,b] iff for each $\in > 0$, there exists $\delta > 0$ such that $U(P, f) - L(P, f) < \in$, for all partitions P with $||P|| < \delta$. (6.5)

(b) Discuss the integrability of the function f defined on [0,1] as follows :

$$f(0) = 0,$$

$$f(x)=\frac{1}{2^n},\text{ if }\frac{1}{2^{n+1}}<x\le\frac{1}{2^n}\ (n=0,1,2,3,..........)$$ (6.5)

(c) Define Gamma function. Prove the duplication formula :

$$\sqrt{\pi}\Gamma(2m)=2^{2m-1}\Gamma(m)\Gamma\left(m+\frac{1}{2}\right).$$ (6.5)

6. (a) Discuss the convergence of the improper integral $\int_0^\infty x^{n-1}e^{-x}\,dx$. (6)

(b) Examine the pointwise and uniform convergence of the sequence of functions $\langle f_n(x) \rangle$ where for each $n \geq 1$,

$$f_n(x) = \frac{nx}{1+n^2x^2},\ 0 \leq x \leq 1. \qquad (6)$$

(c) Prove that the series $\sum_{n=1}^{\infty} \frac{\sin x}{\sqrt{n}}$ is not a fourier series. (6)