[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5274-A **D** **Your Roll No...............**

Unique Paper Code : 235551

Name of the Course : **B.A. Programme**

Name of the Paper : Analysis

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **3** sections.
3. Each section carries **25** marks.
4. Attempt any **two** parts from each question in each section.
5. Marks are indicated against each question.

SECTION I

1. (i) (a) Define supremum and infimum of a set, giving an example. (4½)

(b) State the order completeness property of real numbers using supremum. (2)

(ii) Define an open set. Prove that every open interval is an open set. Which of the following sets are open.

(a) $]2, \infty[$

(b) $[3, 4[$ (6½)

(iii) Prove that the intersection of a finite collection of open sets in $\mathbb{R}$ is open. (6½)

2. (i) Define limit point of a set. Prove that the set $\mathbb{Z}$ of integers has no limit point. (6)

(ii) State the Bolzano Weierstrass Theorem. Prove that its conditions cannot be relaxed. (6)

(iii) Define a uniformly continuous function. Giving an example prove that a continuous function need not be uniformly continuous. (6)

SECTION II

3. (i) Prove that every convergent sequence is bounded. Justify by giving an example that the converse is not true. (4+2½)

(ii) Let $\langle a_n \rangle$ be defined as follows :

$$a_1 = 1,\ a_{n+1} = \frac{4 + 3a_n}{3 + 2a_n},\ n \geq 1$$

Show that $\langle a_n \rangle$ converges. What is the limit of $\langle a_n \rangle$ (6½)

(iii) State Cauchy's Convergence criterion for sequences. Check whether the sequence $\langle S_n \rangle$, where

$$S_n = 1 + \frac{1}{4} + \frac{1}{7} + \dots + \frac{1}{3n - 2}$$

is convergent or not. (6½)

4. (i) Prove that every absolutely convergent series is convergent. What about the converse ? Justify. (6)

(ii) Test for convergence any **two** of the following series :

(a) $$\sum_{n=1}^{\infty} \frac{1.2.3.....n}{7.10....(3n + 4)}$$

(b) $$\sum_{n=1}^{\infty} \left\{ \left(n^3 + 1\right)^{1/3} - n \right\}$$

(c) $\frac{\sin\sqrt{1}}{1} - \frac{\sin\sqrt{2}}{2^{3/2}} + \frac{\sin\sqrt{3}}{3^{3/2}} - \dots$ (6)

(iii) Let $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ be two positive term series such that $\lim_{n\to\infty} \frac{u_n}{v_n} = \ell$ (ℓ is finite and non-zero), then prove that Σu_n and Σv_n converge or diverge together. (6)

SECTION III

5. (i) Define Riemann integrability of a bounded function f on a bounded closed interval [a, b]. Show that the function f defined on [a, b] as

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$$

is not Riemann integrable. (6½)

(ii) Prove that if f is a monotonic function on a bounded closed interval [a, b], then it is integrable on [a, b]. (6½)

(iii) Show that the function f defined on [0,1] as f(x) = [x] is integrable and evaluate $\int_0^3 f(x)dx$. (6½)

6. (i) Test the uniform convergence of the series

$$\sum_{n=1}^{\infty} \frac{\sin nx}{n^p},\ x \in R,\ (p > 1)$$ (6)

(ii) (a) Find a fourier series of the function

$$f(x) = \begin{cases} -1 & -\pi \le x \le 0 \\ 1 & 0 \le x \le \pi \end{cases}$$ (3)

(b) Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{x^n}{\lfloor n}$$ (3)

(iii) Discuss the convergence of the improper integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$ (6)