[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 143 **E** **Your Roll No...............**

Unique Paper Code : 235551

Name of the Course : **B.A. Programme**

Name of the Paper : Analysis

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are 3 sections.
3. Each section carries **25** marks.
4. Attempt any **two** parts from each question in each section.
5. Marks are indicated against each question.

SECTION I

1. (a) Give two examples of the following :

   (i) A set which is bounded above but not bounded below.

   (ii) A set which is bounded below but not bounded above.

   (iii) A set which is neither bounded above nor bounded below. (6)

   (b) Define limit point of a set of real numbers. Find the limit points of the following sets :

   $$B = \left\{1, 2, \frac{1}{2}, 3, \frac{1}{3}, 4, \frac{1}{4}, ...\right\}$$ (6)

(c) Verify the continuity of the following function at $x = 0$:

$$f(x) = e^x \operatorname{sgn}(x + [x]),\ x \neq 0$$

$$f(0) = 0.$$

where sgn denotes the signum function and $[x]$ is the greatest integer $\leq x$. (6)

2. (a) Show that the union of a finite collection of closed sets is a closed set. Give an example to show that the result may not be true for an infinite collection of closed sets. (6.5)

(b) Show that the function f defined as

$$f(x) = \begin{cases} x & \text{if } x \text{ rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

is continuous only at $x = 0$. (6.5)

(c) Prove that $f(x) = x^2$ is not uniformly continuous on $[0, \infty[$ whereas it is uniformly continuous on $]-2, 2[$. (6.5)

SECTION II

3. (a) Show that the sequence $\langle r^n \rangle$ converges to zero if $|r| < 1$. (6.5)

(b) State Cauchy's General Principle of Convergence and show that the sequence $\left\langle 1+\frac{1}{5}+\frac{1}{9}+.....+\frac{1}{4n-3} \right\rangle$ is not convergent. (6.5)

(c) Show that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded. (6.5)

4. (a) Prove that a necessary condition for the convergence of a series $\sum_{n=1}^{\infty} u_n$ is $\lim_{n\to\infty} u_n = 0$. Justify with an example whether the condition is sufficient also for the convergence. (6)

(b) Test for convergence any **two** of the following series : (3+3)

(i) $\frac{1}{3.7}+\frac{1}{4.9}+\frac{1}{5.11}+\frac{1}{6.13}+......$

(ii) $\sum_{n=1}^{\infty}\frac{\sin\left(n^2x+n\right)}{n^3+n},\ x\in R$

(iii) $\sum_{n=1}^{\infty}\frac{1.3.5.....(2n-1)}{2.4.6.....2n}\cdot\frac{1}{n}$

(c) State Leibnitz test for the convergence of an alternating series

$u_1-u_2+u_3-u_4+.....,\ u_n>0\ \forall n.$

Test for absolute convergence and convergence the series $\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+.....$

(6)

SECTION III

5. (a) Let f be a bounded function defined on a closed and bounded interval [a, b]. Define upper sum, lower sum, upper integral, lower integral of f and hence define Riemann integrability of f over [a, b]. Also show that a function f defined as

$f(x) = 2\ \forall\ x \in [0,1]$ is Riemann integrable on [0, 1]. (6)

(b) Examine the convergence of any **two** of the following improper integrals :

(i) $\int_0^1 \frac{\sqrt{x}}{\sin x}dx$

(ii) $\int_0^{\infty} e^{-x}x^2\,dx$

(iii) $\int_1^2 \frac{dx}{(x-1)^3}$ [By Definition] (3+3)

(c) Do any **two** parts : (3+3)

(i) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

(ii) Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^8}} dx = \frac{1}{8} B\left(\frac{3}{8}, \frac{1}{2}\right)$

(iii) Prove that a trigonometric series $\sum_{n=1}^{\infty} \frac{\cos nx + \sin nx}{\sqrt{n}}$ is not a Fourier series.

6. (a) Find Fourier series of function f, where $f(x) = x$, $-\pi \le x \le \pi$. (6.5)

(b) Show that the function f(x) defined on the interval [0, 4] as $f(x) = x[x]$ is integrable on [0, 4], stating clearly the results you are using. (6.5)

(c) Prove that $\langle x^n \rangle$ is not uniformly convergent on [0, 1], but is uniformly convergent on $\left[0, \frac{1}{2}\right]$. (6.5)