

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 144

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Your Roll No.....

Unique Paper Code : 235551

Name of the Course : B.A. Programme

Name of the Paper : Analysis

Semester : V

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are 3 sections.
3. Each section carries 25 marks.
4. Attempt any two parts from each question in each section.
5. Marks are indicated against each question.

SECTION I

1. (a) Define Supremum and Infimum of the set S. Find the Supremum and Infimum of the set :

$$S = \left\{ 1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{2^2}, \dots, 1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^{n-1}}, \dots \right\} \quad (6)$$

- (b) Comment on the following statements :

(i) Can a finite set be open ?

(ii) Can a finite non-empty set be open ?

(iii) Is every infinite set be open ?

Justify the answer by means of an example wherever needed. (6)

P.T.O.

- (c) Prove that if a function f is continuous on a closed and bounded interval $[a, b]$, then it is bounded in $[a, b]$. (6)

2. (a) State Bolzano Weirstrass theorem. Verify the theorem for the set

$$S = \left\{ \frac{1}{n}; n \in \mathbb{N} \right\}. \quad (6.5)$$

- (b) Show that the function f on $[0, 1]$ defined as :

$$f(x) = \frac{1}{3^n} \text{ when } \frac{1}{3^{n+1}} < x \leq \frac{1}{3^n}; (n = 0, 1, 2, 3, \dots)$$

$$f(0) = 0$$

$$\text{is discontinuous at } \frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3}, \dots \quad (6.5)$$

- (c) Show that every uniformly continuous function on an interval is continuous on that interval, but the converse is not true. (6.5)

SECTION II

3. (a) Define convergent sequence. If $\langle a_n \rangle, \langle b_n \rangle$ be two sequences such that

$$\lim_{n \rightarrow \infty} a_n = a, \lim_{n \rightarrow \infty} b_n = b, \text{ then show that } \lim_{n \rightarrow \infty} (a_n b_n) = ab. \quad (6.5)$$

- (b) Show that $\lim_{n \rightarrow \infty} (n)^{1/n} = 1$ and hence show that

$$\lim_{n \rightarrow \infty} \frac{1}{n} (1 + 2^{1/2} + 3^{1/3} + \dots + n^{1/n}) = 1. \quad (6.5)$$

- (c) Define monotonic sequence and show that $\langle a_n \rangle$ defined as $a_1 = 1;$

$$a_{n+1} = \frac{3 + 2a_n}{2 + a_n}; n \geq 1 \text{ is convergent and find its limit.} \quad (6.5)$$

4. (a) Let $\sum_{n=1}^{\infty} u_n$ is a positive term series such that

$$\lim_{n \rightarrow \infty} (u_n)^{1/n} = l.$$

Then prove that

(i) If $l < 1$ then $\sum_{n=1}^{\infty} u_n$ is convergent.

(ii) If $l > 1$ then $\sum_{n=1}^{\infty} u_n$ is divergent. (6)

- (b) Test for convergence any two of the following series :

(i) $\sum_{n=1}^{\infty} \left[\frac{\sqrt{n+1} - \sqrt{n-1}}{n} \right]$

(ii) $\sum_{n=1}^{\infty} \frac{1.3.5. \dots (2n-1)}{2.4.6. \dots 2n} \cdot \frac{1}{2n+1}$

(iii) $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} - \frac{1}{4.5} + \dots$ (3+3)

- (c) State Cauchy's General Principle for convergence of a series. Hence examine the convergence of the series $1 + \frac{1}{5} + \frac{1}{9} + \frac{1}{13} + \dots + \frac{1}{4n-3} + \dots$ (6)

SECTION III

5. (a) Let f be a monotonically increasing function defined on a closed and bounded interval $[a, b]$. Prove that f is Riemann integrable on $[a, b]$. (6)

- (b) Discuss the convergence for any two of the following improper integrals :

(i) $\int_0^1 \frac{dx}{x^n}$ [By Definition]

P.T.O.

$$(ii) \int_0^{\pi} \frac{\sqrt{x}}{\sin x} dx$$

$$(iii) \int_0^{\infty} e^{-nx} x^m dx, \quad m > 0, \quad n > 0 \quad (3+3)$$

$$(c) \text{ Prove that } \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \text{ and hence prove that } \int_0^1 x\sqrt{1-x^4} dx = \frac{\pi}{8}. \quad (6)$$

$$6. (a) \text{ Find Fourier series of function } f, \text{ where } f(x) = |x|, \quad -\pi \leq x \leq \pi. \quad (6.5)$$

(b) Test the pointwise and uniform convergence of $\langle f_n \rangle$ on $[0, 2\pi]$, where

$$f_n(x) = \frac{\sin nx}{\sqrt{n}}, \quad 0 \leq x \leq 2\pi, \quad n \in \mathbb{N}. \quad (6.5)$$

(c) Discuss the Riemann integrability of f over $[0, 1]$, where

$$f(x) = (-1)^n, \quad \frac{1}{n+1} < x < \frac{1}{n}, \quad n \in \mathbb{N}$$

and $f(0) = 0$, clearly stating the result used. (6.5)