[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5443-A **D** **Your Roll No...............**

Unique Paper Code : 235651

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Numerical Analysis and Statistics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.
5. Use of scientific calculator is allowed.

1. (a) (i) Perform three iterations of the bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$.

(ii) If a root of $f(x) = 0$ lies in the interval (a,b), then what is the minimum number of iterations required when the permissible error is $\in$.

(iii) Define rate of convergence of an iterative method. (6)

(b) A real root of the equation

$$f(x) = x^3 - 5x + 1 = 0$$

lies in the interval (0,1). Perform four iterations of the Secant and the Regula-Falsi methods to obtain this root. (6)

(c) Perform four iterations of the Newton-Raphson method to obtain the root of $N^{1/3}$ where N = 17. (6)

2. (a) Consider the system of equations

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Where 'a' is a real constant.

For which values of 'a', the Jacobi and Gauss-seidel methods converge. (6)

(b) Solve the system of equations :

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

Using the Gauss-elimination method with partial-pivoting. (6)

(c) For the following system of equations

$$10x + 4y - 2z = 12$$
$$x - 10y - z = 10$$
$$5x + 2y - 10z = -3$$

(i) Show that the Jacobi iteration scheme converges.

(ii) Starting with $x^{(0)} = 0$, iterate three times. (6)

3. (a) Find the unique polynomial of degree 2 or less, such that $f(1) = 1$, $f(3) = 27$, $f(4) = 64$.

Using newton divided difference interpolation. Estimate $f(2)$. (6½)

(b) For the following data $p(1) = 3$, $p(3) = 31$, $p(4) = 69$, $p(5) = 131$, $p(7) = 351$, $p(10) = 1011$.

Obtain the polynomial using Lagrange interpolating formula. Estimate $p(8.0)$. (6½)

(c) If $f(x) = 1/x$, find the divided difference $f[x_1, x_2, x_3, x_4]$. (6½)

4. (a) Show that arithmetic mean of the regression coefficients is greater than the correlation coefficient. (6)

(b) Fit a parabola of second degree to the following data

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

(6)

(c) Ten competitors in a beauty contest are ranked by three judges as follows :

Competitors

Judges	1	2	3	4	5	6	7	8	9	10
A	6	5	3	10	2	4	9	7	8	1
B	5	8	4	7	10	2	1	6	9	3
C	4	9	8	1	2	3	10	5	7	6

Discuss which pair of judges have the nearest approach to common tastes of beauty. (6)

5. (a) A box contains 4 white and 5 black balls. 3 balls are drawn at random. Find the expected number of white balls drawn. Also find the variance. (6½)

(b) Let X has the pdf $f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$

Consider a random rectangle whose sides are X and (1-X). Determine the expected value of the area of the rectangle. (6½)

(c) Prove that the moment generating function of the sum of a number of independent random variable is equal to the product of their respective moment generating functions. (6½)

6. (a) Comment on the following :

(i) The mean of a binomial distribution is 3 and variance is 4.

(ii) Determine the binomial distribution for which the mean is 4 and variance 3 and find its mode. (6½)

(b) For the poisson distribution with parameter λ show that

$$\mu_{r+1} = r\lambda\mu_{r-1} + \lambda\frac{d\mu_r}{d\lambda}$$

hence find the first four moments about mean. (6½)

(c) Show that the sum and difference of two independent normal variates is also a normal variate. (6½)