This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5463**

Unique Paper Code : **290679** **D**

Name of the Paper : **Application Course — Mathematics for Social Sciences**

Name of the Course : **B.A. (Programme) III**

Semester : **VI**

Duration : **2 Hours** Maximum Marks : **55**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory and carries **15** marks.

Attempt *four* more questions selecting at least *one* question from each Section.

Each question carries **10** marks.

1\. (*i*) Find the value of x such that :

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0.$$

(*ii*) Find all second order partial derivatives of the following function $z = y^2 \log x$.

(*iii*) Solve the differential equation $\frac{dy}{dx} = 2x^2 t$ and find the integral curve that passes through $(t, x) = (1, 2)$.

P.T.O.

(*iv*) Show that :

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2.$$

(*v*) If

$$\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k},$$

$$\vec{b} = -\hat{i} - \hat{j} + \lambda\hat{k},$$

find λ such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal. 5×3=15

Section I

2. (*i*) Find the area of the region bounded by the curves $y = x^2$ and $y^2 = x$. 5

(*ii*) The marginal cost of production is found to be MC = $2000 - 40x + 3x^2$, where x is the number of units produced. The fixed cost of production is Rs. 18,000. Find the total cost function. 5

3. (*i*) Find the angle between the vectors :

$$\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k} \text{ and}$$

$$\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}.$$ 5

(*ii*) Solve the following differential equation :

$$(ax + hy + g)dx + (hx + by + f)dy = 0.$$ 5

Section II

4. (*i*) Show that matrix :

$$A = \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

is orthogonal. 5

(*ii*) Let

$$A = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}.$$

Find the characteristic equation of A. Verify Cayley-Hamilton theorem for A. 5

5. (*i*) Solve the following system of linear equations by Cramer's rule :

$$x - 4y - z = 11;$$

$$2x - 5y + 2z = 39;$$

$$-3x + 2y + z = 1.$$ 5

(*ii*) Is the following system of equations consistent :

$$x - 3y + 4z = 3;$$

$$2x - 5y + 7z = 6;$$

$$3x - 8y + 11z = 11.$$ 5

Section III

6. (*i*) The production function of firm is given by :

$$Q = 20LK - 2L^2 - 4K^2 + 800,$$

show that :

$$L\frac{\partial Q}{\partial L} + K\frac{\partial Q}{\partial K} = 2Q.$$ 5

(*ii*) Find the angle between vectors :

$$\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k} \text{ and}$$

$$\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}.$$ 5

7. (*i*) Use graphical method to solve the following linear programming problem :

Minimize : $z = 18x + 10y$

Subject to constraints : $4x + y \geq 20;$

$2x + 3y \geq 30;$

$x, y \geq 0.$ 5

(*ii*) The joint cost function of a firm producing two products is given by :

$$C(x, y) = 6x^2 - 9x - 3xy - 7y + 5y^2 + 20,$$

where x and y denote their units. Find the values of x and y that minimize $C(x, y)$. Also find the minimum cost. 5