[This question paper contains 4 printed pages.]

**Sr. No. of Question Paper : 294** **E** **Your Roll No................**

Unique Paper Code : 235651

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Numerical Analysis and Statistics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.
5. Use of scientific calculator is allowed.

1. (a) Approximate $\sqrt[3]{13}$ to three decimal places by applying the bisection method to the equation

$$x^3 - 13 = 0 \quad (6)$$

(b) Perform 4 iterations of the Newton-Raphson method to determine the root of the equation $x^5 + 2x - 1 = 0$, lying in the interval (0,1). (6)

(c) Find the smallest positive root of the equation

$x^3 - x^2 - x + 1 = 0$ correct to three decimal places using the secant method. (6)

2. (a) For the following system of equations

$$2x_1 - x_2 + 0x_3 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$0x_1 - x_2 + 2x_3 = 1$$

(i) Show that the Gauss-Siedel iteration scheme converges.

(ii) Starting with $x^{(0)} = 0$, iterate three times. (6)

(b) Solve the following system of equations

$$\begin{bmatrix} 2 & 2 & 1 \\ 4 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

using the Gauss elimination method with partial pivoting. (6)

(c) Solve :

$$4x + y + 2z = 4$$

$$3x + 5y + z = 7$$

$$x + y + 3z = 3$$

Using the Gauss-Jacobi method with the initial approximation as $x^{(0)} = (0,0,0)$, perform three iterations. (6)

3. (a) Find the unique polynomial of degree 2 or less, such that f(1) = 1, f(3) = 27, f(4) = 64. Using newton divided difference interpolation. Estimate f(2). (6½)

(b) For the following data p(−1) = −2, p(1) = 0, p(4) = 63, p(7) = 342

Obtain the polynomial using Lagrange interpolating formula. Estimate p(0). (6½)

(c) If f(x) = 1/x, find the divided difference $f[x_1, x_2, x_3, x_4, x_5]$. (6½)

4. (a) The variables X and Y are connected by the equation $aX + bY + c = 0$. Show that the correlation between them is $-1$ if the sign of a and b are all alike and +1 if they are different. (6)

(b) Obtain the equations of the lines of regression of the following data

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Y | 9 | 8 | 10 | 12 | 11 | 13 | 14 | 16 | 15 |

(6)

(c) For a discrete distribution prove that the mean deviation about the mean $\bar{x}$ can be written in the

Form $\frac{2}{N}\left[\bar{x}\sum_{x_i<\bar{x}} f_i - \sum_{x_i<\bar{x}} f_i x_i\right]$. (6)

5. (a) Three urns respectively contains 3G, 2W; 5G, 6W; 2G, 4W balls. one ball is drawn from each urn. Find the expected number of white balls drawn. (6½)

(b) Suppose that X is a random variable with $E[X] = 10$ and $Var[X] = 25$. Find the positive numbers a and b such that $Y = aX - b$ has mean 0 and variance 1. (6½)

(c) Let the random variable X has the pdf

$$f(x) = q^{x-1}p, \ x = 1,2,3,........$$

Find the moment generating function and hence mean and variance. (6½)

6. (a) Find the mode of the Binomial distribution. (6½)

(b) Show that in a Poisson distribution with unit mean, E[|X – 1|] is 2/e times the standard deviation. (6½)

(c) Show that for the normal distribution, all odd order moments about the mean vanish and the even order moments about the mean are equal to

$\mu_{2n} = 1.3.5.......(2n-1)\sigma^{2n}$ (6½)