[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 295 **E** **Your Roll No................**

Unique Paper Code : 235651

Name of the Course : **B.A. (Prog.)**

Name of the Paper : Numerical Analysis and Statistics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt **two** parts from each question.
4. **All** questions are compulsory.
5. Use of scientific calculator is allowed.

1. (a) The smallest positive root of the following equation is to be obtained

$$f(x) = \log(1 + x) - \cos x = 0$$

(i) Find an interval of unit length which contains the root.

(ii) Perform 4 iterations of the bisection method. (6)

(b) Perform 4 iterations of the Regula-Falsi method to determine the root of the equation

$x^3 - x - 4 = 0$, lying in the interval (1.5,2). (6)

(c) Find the iterative method based on the Newton-Raphson method for finding $\sqrt{N}$, where N is a positive real number. Apply the method to N = 24 to obtain the result correct to two decimal places. (6)

2. (a) Solve :

$$9x_1 + 2x_2 + 4x_3 = 20$$

$$x_1 + 10x_2 + 4x_3 = 6$$

$$2x_1 - 4x_2 + 10x_3 = -15$$

using the Gauss-Seidel method with the initial approximation $x^{(0)} = (0,0,0)$, Perform three iterations. (6)

(b) Solve the following system of equations using the Gauss-elimination method with partial pivoting :

$$x_1 + x_2 - x_3 = 2$$

$$2x_1 + 3x_2 + 5x_3 = -3$$

$$3x_1 + 2x_2 - 3x_3 = 6$$ (6)

(c) For the following system of equations

$$x + 2y - 2z = 11$$

$$x + y + z = 0$$

$$2x + 2y + z = 3$$

(i) Show that the Jacobi iteration scheme converges.

(ii) Starting with $x^{(0)} = 0$, iterate three times. (6)

3. (a) Find the unique polynomial of degree 2 or less, such that $f(0) = 1$, $f(1) = 3$, $f(3) = 55$.

Using newton divided difference interpolation. Estimate $f(2)$. (6½)

(b) For the following data $p(-1) = -5$, $p(2) = 13$, $p(4) = 255$, $p(5) = 625$

Obtain the polynomial using Lagrange interpolating formula. Estimate $p(3)$. (6½)

(c) If $f(x) = 1/x^3$, find the divided difference $f[x_1, x_2, x_3, x_4, x_5]$. (6½)

4. (a) If X and Y are random variables with correlation coefficient p between them, then show that

$$U = x \sin \alpha + y \cos \alpha$$

$$V = y \sin \alpha - x \cos \alpha$$

are uncorrelated if

$$\tan 2\alpha = \frac{2p\sigma_x\sigma_y}{\sigma_y^2 - \sigma_x^2}$$ (6)

(b) Show that for discrete distribution $\beta_2 > 1$. (6)

(c) Ten students got the following percentage of marks

Students	1	2	3	4	5	6	7	8	9	10
Economics	78	36	98	25	75	82	90	62	65	39
Mathematics	84	51	91	60	68	62	86	58	53	47

Calculate the rank correlation coefficient. (6)

5. (a) find the mean and variance, if they exist, of the distribution

$$f(x) = \begin{cases} \dfrac{3!}{x!(3-x)!}\left(\dfrac{1}{2}\right)^3, & x = 0,1,2,3 \\ 0, & \text{elsewhere} \end{cases}$$ (6½)

(b) If the variance of the random variable X exists, show that

$$E[X^2] \geq (E(X))^2$$ (6½)

(c) For the following distribution compute $P[\mu - 2\sigma < X < \mu + 2\sigma]$

$f(x) = 6x(1-x)$ for $0 < x < 1$ and 0 elsewhere. (6½)

6. (a) Show that poisson distribution is a limiting case of a binomial distribution. (6½)

(b) If the random variable X has a geometric distribution with parameter p then show that

$$E[X] = \frac{q}{p}, \quad Var[X] = \frac{q}{p^2} \qquad (6½)$$

and $M_X(t) = \frac{p}{1-qe^t}$

(c) For a certain normal distribution ,the first moment about 10 is 40 and the fourth moment about 50 is 48. What is the mean and standard deviation of the distribution. (6½)