This question paper contains **4+1** printed pages]

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* the questions as per the directions questionwise.

1. Attempt any *four* parts :

    (*a*) Find the equation of a line which passes through the point (3, 5, 2) and moves in the direction (1, 1, 1) in parametric form.   6

    (*b*) Show that the set {(1, 0, 1, 0), (0, 1, −1, 2), (0, 2, 2, 1), (1, 0, 0, 1)} is a basis for R^4.   6

    (*c*) Find the matrix representation for counterclockwise rotation of the plane about origin through 90°.   6

    (*d*) Let $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$. Find the eigenvalues and the corresponding eigenvectors of the matrix A.   6

(e)   Find an eigenspace of the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$.   6

2.   Attempt any *four* parts :

(a)   Find the general term for the following sequences :   6

(i) $\dfrac{2}{3}, \dfrac{5}{6}, \dfrac{10}{11}, \dfrac{17}{18}, \dots\dots\dots\dots$

(ii) $\dfrac{1}{2}, \dfrac{3}{5}, \dfrac{5}{10}, \dfrac{7}{17}, \dots\dots\dots\dots$

(iii) $\dfrac{3}{2}, \dfrac{5}{9}, \dfrac{7}{28}, \dfrac{9}{65}, \dots\dots\dots\dots$

(b)   Show that the sequence $<r^n>$ converges to zero if $|r| < 1$.   6

(c)   Show that the series $\Sigma \dfrac{1}{n}$ does not converge.   6

(d)   State limit comparison test for positive term series. Show that the series :   6

$$\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \dots\dots\dots$$

is convergent.

(e)   State ratio test for a positive term series. Use it to discuss the convergence of the series

$\Sigma \dfrac{1}{n^n}$.   6

3.   Attempt any *two* parts :

(a)   Define Fibonacci numbers. Write a 'SPARKS' program to print the value of the nth Fibonacci number Fn.   4

(b) Find the greatest common divisor of the pair :   4

(85,119).

(c) Define 'Big Oh' notation. Show that :

$$f(n) = 60n^2 + 5n + 1 = \theta(n^2).$$

4. Attempt any *three* parts :

(a) (i) Define in-degree and out-degree of a vertex in a directed graph. Give example for each.

(ii) Define a tree. Give an example of a tree.   4

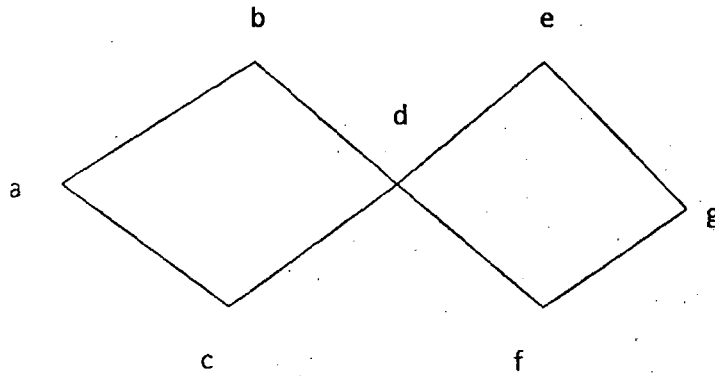(b) Draw graphs with the following adjacency matrices :   4

(i)
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(ii)
$$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}.$$
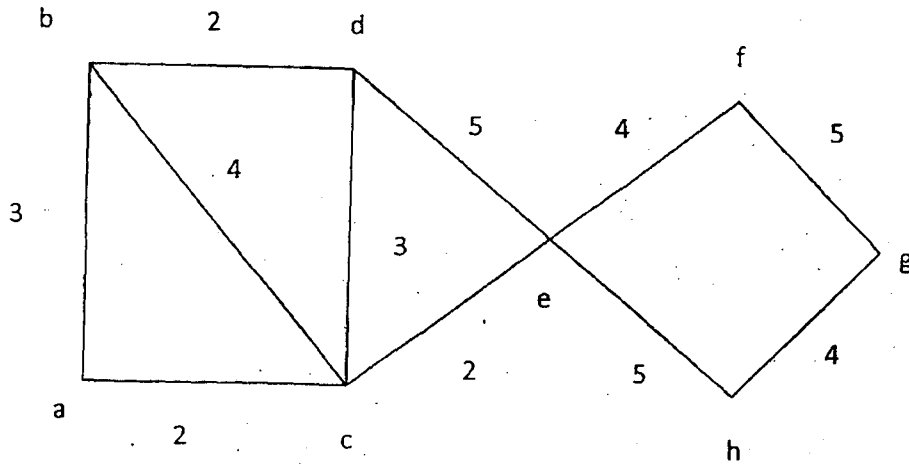
(*c*) Find the Depth-first search spanning tree of the graph : 4

(*d*) Use Prim's algorithm to find a minimal spanning for the following graph : 4

5. Attempt any *two* parts :

(*a*) Define the following terms : 3½

(*i*) Saddle point

(*ii*) Pure strategy.

(b)   A and B play game in which each has three coins, a 5*p*, a 10*p* and a 20*p*. Each player selects a coin without the knowledge of the other choice. If the sum of the coins is an odd amount A wins B's coins, if the sum is even B wins A's coins. Find the best strategy for each player and the value of the game.   3½

(c)   Solve the game whose pay-off matrix is :   3½

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}.$$