Unique Paper Code : 235381

Name of the Paper : **Mathematics**

Name of the Course : **B.Com.(Hons.)**

Semester : **III**

Duration : **3 Hours**

Maximum Marks : 75

E

Q1. Attempt any *four* parts:

a) Find the equation of a line which passes through $(-1, 2, 4)$ that is parallel to $3i - 4j + k$ in symmetric form.   6

b) Determine whether the set of vectors

$$S = \{(2, 1, 4), (1, -1, 2), (3, 1, -2)\}$$

form a basis for $\mathbb{R}^3$.   6

c) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that:
$$T(1, 0, 0) = (1, 2, -1)$$
$$T(0, 1, 0) = (1, 0, 2)$$
$$T(0, 0, 1) = (1, 1, 3)$$ 6

Find $T(2, -1, 3)$

d) Find the standard matrix representing the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined as anti-clockwise rotation through $\frac{\Pi}{3}$.   6

e) Find the characteristic equation of the matrix:

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 3 \end{bmatrix}$$ 6

Q2. Attempt any *four* parts:

a) Find the first five terms of the following sequences:
i. $a_1 = 2, a_{n+1} = \frac{(-1)^{n+1} a_n}{2}$
ii. $y_1 = 2, y_{n+1} = \frac{1}{2}(y_n + \frac{2}{y_n})$ 6

b) Show that $\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n = e^x$ 6

c) Determine whether the series:

$$\sum_{k=1}^{\infty} \left(\frac{3}{4^k} - \frac{2}{5^{k-1}}\right)$$ 6

Converges or diverges?

d) Show that the series:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

6

converges to 1.

e) State limit comparison test for positive term series and use it to determine whether the following series converges:

$$1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \cdots$$

6

Q3. Attempt any *two* parts:

a) Write the general forms of the following statements of 'SPARKS'

    i.   Assignment
    ii.   If then else
    iii. Case
    iv. While

4

b) Find the greatest common division of
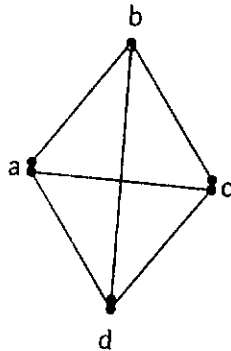
(14, 36, 119)

4

c) Define 'Big Oh' notation. Show that:

$$f(n) = 8n^4 + 6n^2 - 5n - 5 = O(n^4)$$

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Q4. Attempt any *three* parts:

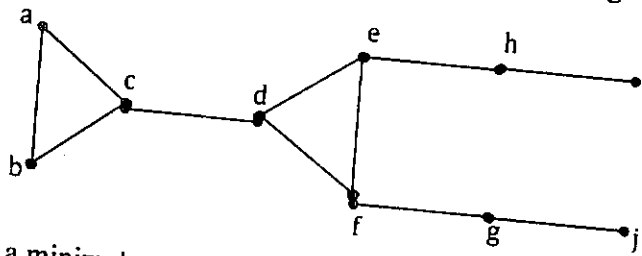a) Define a sub-graph of a graph G. Draw three sub-graphs of the following graph G:

4

b) Find the graph represented by the following adjacency matrix:

$$\begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$
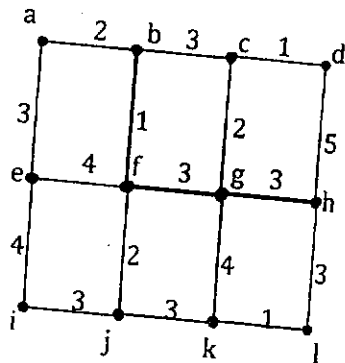
4

c) Use Breadth-First-Search algorithm to find a spanning tree for the graph:

4

d) Find a minimal spanning tree for the following graph:

4

Q5. Attempt any *two* parts:

a) Define the following terms:

i. Zero - Sum Game
ii. Pure Strategy

3.5

b) Two players A and B match coins. A wins 8 units of value if two coins turn both heads and 1 unit of value if two coins turn both tails. B wins 3 units of value when the two coins do not match. Given the choice of being A or B, which one would you choose and what would be your strategy?

3.5

c) Solve the following game using the notion of Dominance whose pay-off matrix is:

$$\begin{bmatrix} 1 & 3 & 2 & 2 \\ 7 & -5 & 1 & 2 \\ 4 & -1 & 2 & 2 \\ 3 & -2 & 2 & 2 \end{bmatrix}$$

3.5