This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5529**

Unique Paper Code : **235381** **F**

Name of the Paper : **Mathematics (CH-3.5)**

Name of the Course : **B.Com. (Hons.)**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* the questions as per the directions questionwise.

1. Attempt any *four* parts :

(*a*) Prove that the function $T : \mathbf{R}^2 \rightarrow \mathbf{R}^3$ defined by

$$T(x, y) = (x, y, 1)$$

is not a linear transformation. 6

(*b*) Determine whether the set of vectors

$$S = \{(1, 2, 3), (0, 1, 1), (1, 0, 0)\}$$

forms a basis for $\mathbf{R}^3$? 6

(*c*) Find the standard matrix for the linear transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ defined by 6

$$T(x, y, z) = (y, z, x).$$

P.T.O.

(*d*) Find the eigenvalues of the matrix : 6

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

(*e*) Find the eigenspace of the matrix : 6

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

2. Attempt any *four* parts :

(*a*) Find the general term of the following sequences : 6

(*i*) $\frac{2}{5}, \frac{4}{11}, \frac{6}{17}, \frac{8}{23}, \ldots\ldots\ldots\ldots$

(*ii*) $1, \frac{-1}{3}, \frac{1}{9}, \frac{-1}{27}, \ldots\ldots\ldots\ldots$

(*iii*) $\frac{2}{3}, \frac{5}{6}, \frac{10}{11}, \frac{17}{18}, \ldots\ldots\ldots\ldots$

(*b*) Find the limit of the following sequence : 6

$$\left\{\frac{1 + 2 + 3 + \ldots\ldots + n}{n^2}\right\}_{n=1}^{\infty}$$

(*c*) Prove that the series :

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots\ldots\ldots\ldots$$

is not convergent. 6

(*d*) Find the sum of the series Σu_n, where 6

$$u_n = \frac{5}{2^n} + \frac{1}{3^n}.$$

(*e*) State limit comparison test for positive term series and use it to determine whether the following series converges : 6

$$\frac{1}{3} + \frac{1}{5} + \frac{1}{9} + \frac{1}{17} + \ldots\ldots\ldots\ldots\ldots .$$

3. Attempt any *two* parts :

(*a*) Define Fibonacci numbers. Write a 'SPARKS' program to print the value of the *n*th Fibonacci number F_n. 4

(*b*) Find the greatest common divisor of 414 and 662. 4

(*c*) Define 'Big Oh' notation. Show that : 4

$$f(n) = 60\ n^2 + 5n + 1 = O(n^2).$$

4. Attempt any *three* parts :

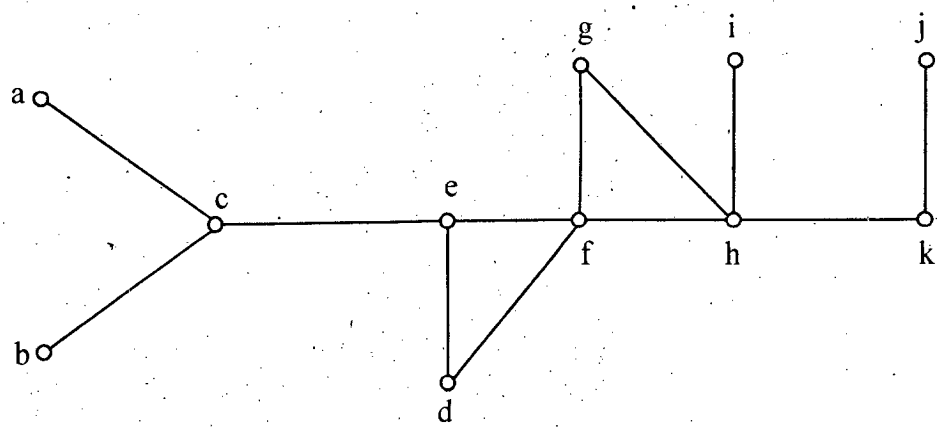
(*a*) What do you mean by Konigsberg Bridge problem ? Explain it. 4

(*b*) Find the graph for the following adjacency matrices : 4

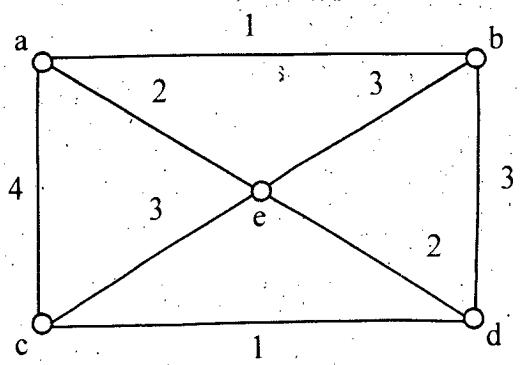
(*i*)
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(*ii*) $\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$

(*c*) Use Breadth First Search algorithm to find a spanning tree for the graph : 4



(*d*) Find a minimal spanning tree for the following graph : 4



5. Attempt any *two* parts :

(*a*) Define the following terms :

(*i*) Mixed strategy

(*ii*) Two-person zero-sum game. 3.5

(*b*) Two players A and B match coins. Suppose A wins one unit of value when there are two heads, wins nothing when there are two tails and loses $\frac{1}{2}$ unit of value when there is one head and one tail. Determine the optimum strategies for the players and value of the game. 3.5

(*c*) For what value of α, the game with the following pay-off matrix is strictly determinable ? 3.5

$$\begin{bmatrix} \alpha & 6 & 2 \\ -1 & \alpha & -7 \\ -2 & 4 & \alpha \end{bmatrix}.$$