[This question paper contains 4 printed pages.]

4705 *Your Roll No.* ....

B.Sc. (G)/I **AS**

MATHEMATICAL SCIENCES (STATISTICS)

Paper II – Probability

Time : 3 hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** *questions in all.*
Question No. **1** *is compulsory and choose* **five** *from the remaining questions.*

1. (a) State which of the following statements are true or false. In case of a false statement, give the correct statement :

(i) The arithmetic mean of the numbers $1, 2, --- n$, each with equal frequency f is $\frac{n(n+1)}{2f}$.

(ii) If A and B are independent events then $P(A \cup B) = P(A) + P(B)$.

(iii) If $P(A|B) < P(A)$ then $P(B|A) < P(B)$

(iv) For two random variables X and Y, $E(XY) = E(X).E(Y)$.

(b) Find the mode of the distribution given below :

$$f(x) = \left(\frac{1}{2}\right)^{x+1}; \ x = 0, 1, 2, \ldots .$$

(c) Let X be a random variable with mean μ and variance σ^2. Show that $E(X-b)^2$ is minimum when $b = \mu$.

(d) Let X be a random variable with distribution function $F(x) = K(1-e^{-x})^2; \ x > 0$.

Find the constant K.

(e) (i) State the relationship between $M_X(t)$ and $E(X^i)$.

(ii) Show that $Var(CX) = C^2 \, Var(X)$.

(2,1,1½,1½,2)

2. (a) For two events A and B with $P(B) \neq 1$, prove that

$$P(A|\bar{B}) = \frac{P(A) - P(A \cap B)}{1 - P(B)}.$$

Hence deduce that $P(A \cap B) \geq P(A) + P(B) - 1$.

(b) If a machine is correctly set up it will produce 90% acceptable items. If it is incorrectly set up it will produce 30% acceptable items. Past experience shows that 80% of set ups are correctly done. If after a certain setup, first item produced is acceptably what is the probability that the machine is correctly set up ?

(c) In a random arrangement of the letters of the word MATHEMATICS, find the probability that all the vowels come together. (2,2½,1½)

3. (a) Two random variables X and Y have the following joint probability density function :

$$f(x, y) = K(u - x - y);\ \ 0 \le x \le 2,\ \ 0 \le y \le 2.$$

Find the constant K. Also find the marginal density functions of X and Y and conditional density function of X given Y.

(b) The probability mass function of a random variable X is zero except at the points X = 0, 1, 2. At these points it has values $P(0) = 3C^3$, $P(1) = 4C - 10C^2$ and $P(2) = 5C - 1$ for some $C > 0$.

Determine the value of C and hence find mean and variance of X. (3,3)

4. (a) A random variable X has the p.d.f.

$$f(x) = 2x\ ;\quad 0 < x < 1.$$

Find $P\left(X < \frac{1}{2}\right)$, $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$, $P\left(X > \frac{3}{4} \middle| X > \frac{1}{2}\right)$ and median of the distribution.

(b) Three coins are tossed. Let X denote the number of heads on the first two coins and Y denote the number of tails on the last two coins. Find the joint distribution of X and Y. Also find mean and variance of [illegible]. (3,3)

5. (a) Obtain the m.g.f. of the random variable X having the p.d.f.

$$f(x) = \begin{cases} x & 0 \le x < 1 \\ 2-x & 1 \le x < 2. \end{cases}$$

Determine $\mu_1{}^1$, μ_2, μ_3 and μ_4.

(b) If X and Y are two random variables such that $Y \le X$, then show that $E(Y) \le E(X)$, provided the expectations exist.

(c) If X and Y are independent random variables with $E(X) = \alpha$, $E(X^2) = \beta$ and $E(Y^K) = a_K$, $K = 1, 2, 3, 4$, find $E(XY + Y^2)^2$. (3,1½,1½)

6. (a) Let X have the p.d.f. : $f(x) = \frac{1}{2\sqrt{3}}$; $-\sqrt{3} < x < \sqrt{3}$.

Find the actual probability $P\left(|X - \mu| \ge \frac{3}{2}\sigma\right)$ and compare it with the upper bound obtained by Chebyshev's inequality.

(b) A player tosses 3 fair coins. He wins Rs. 8, if 3 heads occur; Rs. 3 if two heads occur and Rs. 1 if one head occurs. If his expected gain is zero, how much should he lose, if no heads occur ?

(3,3)

7. (a) State and prove Lindeberg-Levy central limit theorem for i.i.d. random variables.

(b) The p.d.f. of a variable Y is $f(y) = \frac{1}{2}$; $-1 \le y \le 1$. Show that the sequence $\{X_n\}$ of uncorrelated random variables where $X_n = \sin(n\pi Y)$, $n = 1, 2, \ldots$ satisfies WLLN (3,3)

(200)****