[This question paper contains 4 printed pages.]

4685 *Your Roll No.* ....

B.Sc. (G) / I / NS **AS**

MATHEMATICAL SCIENCES

Paper I – Essentials for Operational Research (Statistics)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any **Five** *questions.*

1. (a) The following table shows the distribution of 100 families according to their expenditure per week. Number of families corresponding to expenditure in thousand rupees groups Rs. (10-20) and Rs. (20-30) are missing from the table. The median and mode are given to be Rs. 25 and Rs. 24 respectively. Calculate the missing frequencies.

Expenditure :	0-10	10-20	20-30
No. of families :	14	?	27

Expenditure :	30-40	40-50
No. of families :	?	15

(6)

(b) Show that standard deviation is the minimum value of root mean square deviation. (5)

P.T.O.

2. (a) Define the raw and central moments of a frequency distribution. Obtain the relation between the central moments of order r in terms of the raw moments. (6)

   (b) A can hit a target 3 times in 5 shots, B 2 times in 5 shots and C 3 times in 4 shots. Find the probability of the target being hit at all when all of them try. (5)

3. (a) State and prove Baye's theorem. (6)

   (b) Define moment generating function. Show that moment generating function of the sum of a number of independent random variables is equal to the product of their respective moment generating functions. (5)

4. (a) The following is the recurrence relation for moments of a Binomial distribution with parameters n and p;

$$\mu_{r+1} = pq\left(nr\mu_{r-1} + \frac{d}{dp}\mu_r\right)$$

   Find the value of β_1 and β_2. Also show that $\beta_1 \to 0$ and $\beta_2 \to 3$ as $n \to \infty$. (6)

   (b) If X is a Poisson variate with parameter λ such that

$$P(X = 2) = 9\ P(X = 4) + 90\ P(X = 6)$$

   Find (i) λ, the mean of X and (ii) the Karl Pearson's co-efficient of skewness. (5)

5. (a) Show that the sum and difference of any two independent normal variates is again a normal variate. (5)

(b) X is normally distributed with mean 12 and variance 16.

Find : (i) the probability of $0 \le X \le 12$ and (ii) x_0^1 and x_1^1 when $P(x_0^1 \le X \le x_1^1) = 0.50$ and $P(X \ge x_1^1) = 0.25$. (6)

6. (a) Explain the principle of least squares. Use it to fit a straight line to the following data :

X :	1	2	3	4	6	8
Y :	2.4	3	3.6	4	5	6

(6)

(b) Why are there two lines of regression ? Show that regression co-efficients are independent of the change in origin but not scale. (5)

7. (a) Define a Markov Chain. If $\{X_n, n \ge 0\}$ is a Markov chain with three states 0, 1, 2 and with transition matrix :

$$\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

Find

(i) $P\{X_2 = 2 | X = 1\}$

(ii) $P\{X_2 = 2,\ X_1 = 1 \mid X_0 = 2\}$

(iii) $P\{X_2 = 2,\ X_1 = 1,\ X_0 = 2\}$ (6)

(b) Consider an inventory situation in a manufacturing concern. If the number of sales per day is Poisson with mean 5, then generate 20 days of sales by Monte Carlo method. (5)

(Use the following random numbers

10, 48, 01, 50, 15, 08, 53, 60, 22, 16,
08, 09, 16, 01, 04, 21, 15, 21, 01, 03)