[This question paper contains 4 printed pages.]

4692 *Your Roll No.* ....

B.Sc. (G)/I **AS**

MATHEMATICS – Paper II

Calculus

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt any **Two** *parts from each question.*
Questions **1** *to* **5** *carry* **9** *marks each.*
Q. **6** *carries* **10** *marks.*

1. (a) Let the function f be defined as

$$f(x) = \begin{cases} 7x - 2, & x \le 1 \\ Kx^2, & x > 1 \end{cases}$$

Find a value for the Constant K, for which the function f is continuous everywhere. (4½)

(b) Discuss the continuity of the function

$$f(x) = \begin{cases} (x-a)\sin\dfrac{1}{x-a}, & \text{for } x \ne a \\ 5, & \text{for } x = a \end{cases}$$

at x = a. Also discus the kind of discontinuity. (4½)

P.T.O.

(c) Discuss the derivability of the function

$$f(x) = |x| + |x - 1|$$

at $x = 0$ and $x = 1$. (4½)

2. (a) Find the nth derivative of $\tan^{-1}\left(\frac{x}{a}\right)$. (4½)

(b) If $y = e^{a \sin^{-1} x}$, prove that

$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0.$
Where y_n denotes the nth derivative of y with respect to x. (4½)

(c) If $u = \tan^{-1}\frac{x^3 + y^3}{x + y}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u \cdot$$ (4½)

3. (a) If the straight line

$$P = x\cos\alpha + y\sin\alpha$$

touches the curve

$$\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1$$

Prove that

$(a\cos\alpha)^n + (b\sin\alpha)^n = p^n.$ (4½)

(b) If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle θ with the x-axis, show that its equation is

$$y \cos \theta - x \sin \theta = a \cos 2\theta \qquad (4\tfrac{1}{2})$$

(c) Find the points of intersection of the circle $x^2 + y^2 = \sqrt{2}\, a^2$ and the rectangular hyperbola $x^2 - y^2 = a^2$. Also find the angle of intersection at any one of the points. (4½)

4. (a) Find the asymptotes of the curve

$$(x + y)^2 \, (x + 2y + 2) = x + 9y - 2. \qquad (4\tfrac{1}{2})$$

(b) Trace the curve :

$$x^3 + y^3 = a^2 x \qquad (4\tfrac{1}{2})$$

(c) Prove that for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ the radius of}$$

curvature 'p' is given by

$$p = \frac{a^2 b^2}{p^3},$$

p being the perpendicular from the centre upon the tangent at (x, y). (4½)

5. (a) Evaluate $\int \sqrt{\frac{1-x}{1+x}} \frac{dx}{x}$. (4½)

(b) Prove that

$$\int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin x \cos x} = \frac{\pi}{3\sqrt{3}}$$ (4½)

(c) If $u_n = \int \text{Cos } n\theta \ \text{Cosec}\theta \, d\theta$, prove that

$$u_n - u_{n-2} = \frac{2\cos(n-1)\theta}{n-1}$$

Hence or otherwise prove that

$$\int_0^{\pi/2} \frac{\sin 3\theta \sin 5\theta}{\sin\theta} d\theta = \frac{71}{105}$$ (4½)

6. (a) Find the entire length of the astroid

$x = a\cos^3 t, \quad y = a\sin^3 t.$ (5)

(b) Find the area of the curve

$x^2(x^2 + y^2) = a^2(x^2 - y^2).$ (5)

(c) Find the volume formed by the revolution of the loop of the curve

$$y^2(1 + x) = x^2(1 - x)$$

about the x-axis. (5)