

[This question paper contains 4 printed pages.]

1461-A

Your Roll No.

B.A./B.Sc. (Hons.)/I A

MATHEMATICS – Unit-II

(Algebra-I)

(Admissions of 2008 and before)

Time : 2 Hours

Maximum Marks : 38

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*Attempt any two parts from each Section.
Marks are indicated against each question.*

SECTION I

1. (a) (i) If $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$, find the value of x_1, x_2, x_3, \dots upto ∞ .

- (ii) If α, β, γ are the roots of the equation

$$x^3 + ax^2 + bx + a = 0,$$

where a and b are real, show that

$$\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = n\pi$$

except when $b = 1$. (2,2½)

- (b) Sum the series

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \dots + \text{upto } n \text{ term.}$$

(4½)

P.T.O.

- (c) Using De-Moivre's theorem, solve the equation
 $z^5 + z^4 + z^3 + z^2 + z + 1 = 0.$ (4½)

SECTION II

2. (a) Prove that an elementary row operation on the product of two matrices is equivalent to the same elementary row operation on the pre-factor. (5)
- (b) Define the rank of a matrix. Reduce the matrix

$$A = \begin{pmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{pmatrix}$$

to normal form using elementary operations and hence determine its rank. (5)

- (c) (i) State Cayley Hamilton theorem. Verify that the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix}$$

satisfies its characteristic equation. (2½)

- (ii) If A and P are square matrices of the same order and P is invertible, show that A and $P^{-1}AP$ have the same characteristic roots.

(2½)

SECTION III

3. (a) For what values of λ does the following system of equations have a solution

$$\begin{aligned}x + y + z &= 1 \\x + 2y + 4z &= \lambda \\x + 4y + 10z &= \lambda^2\end{aligned}$$

Also find the solution in each case. (5)

- (b) (i) Solve the equation

$$x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$$

given that the sum of two roots is equal to the sum of the other two roots. (2½)

- (ii) If α, β, γ are the roots (all non-zero) of the equation

$$x^3 - px^2 + qx - r = 0$$

find the value of (i) $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$,
(ii) $\sum \alpha/\beta$. (2½)

- (c) (i) Solve the equation

$$x^4 - 12x^3 + 49x^2 - 78x + 40 = 0$$

by removing its second term. (2½)

(ii) If $\alpha + \beta + \gamma = 1$

$$\alpha^2 + \beta^2 + \gamma^2 = 2$$

$$\alpha^3 + \beta^3 + \gamma^3 = 3$$

find the value of $\alpha^4 + \beta^4 + \gamma^4$. (2½)

SECTION IV

4. (a) (i) If
- p
- is a prime number and
- a, b
- are integers such that
- p
- divides
- ab
- ,
- p
- divides
- a
- or
- p
- divides
- b
- . (2½)

- (ii) If
- $\text{g.c.d.}(a, b) = 1$
- , show that

$$\text{g.c.d.}(a^n, b^n) = 1 \text{ and } \text{g.c.d.}(a+b, a-b) = 1 \text{ or } 2. \quad (2)$$

- (b) (i) Solve the Congruence
- $8x \equiv 5 \pmod{27}$
- . (2½)

(ii) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$, $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}$

Compute $\sigma^{-1}\rho\sigma$. (2)

(c) (i) Express $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 5 & 2 & 8 & 9 & 4 & 1 & 6 & 10 & 7 \end{pmatrix}$

as a product of disjoint cycles. (2½)

- (ii) Prove that in
- S_n
- , the number of distinct
- r
- cycles (
- $r \leq n$
-) is
- $\frac{n!}{r(n-r)}$
- . (2)