[This question paper contains 6 printed pages.]

565 *Your Roll No.* ....

B.Sc. Prog./B.Sc. (Hons.)/I **A**

M.A.-107-B – MATHEMATICS

(For Life Sciences)

(Admissions of 2008 & onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are **three** *Sections in this question paper. Attempt any* **two** *questions from each Section. Students are allowed to use calculators.*

SECTION I

1. (a) Let P_1 and P_2 be two points in a plane. Let A_1, A_2 be the sets of all straight lines through P_1 and P_2 respectively. Find $A_1 \cap A_2$. (4)

(b) Find the product set of $A = \{0, 2, 4\}$ and $B = \{0, 1\}$. Show that commutative law does not hold for the cross product. (4)

(c) A linear function G(t) assume the value $G_1 = 88.3$ mg at time instant $t_1 = 14$ seconds and the value $G_2 = 89.6$ mg at $t_2 = 39$ seconds. Find the growth rate and establish the linear function. (4½)

2. (a) For $x = -1, 0, 1$ the quantity $y = 8 - 3x + x^2$ takes on the values 12, 8, 6 respectively. Using first and second differences only, find the functional values for $x = 2, 3, 4$. (4½)

(b) Let x be a natural number which is also a multiple of 3 and y be the remainder after dividing x by 5. Is y a periodic function of x ? If yes, what is the period ? Plot a graph of the function. (4)

(c) A certain culture of bacteria grows at a rate proportional to the number present. It is found that the number doubles in 4 hours. Find how many bacteria may be expected at the end of 24 hours. (4)

3. (a) Discuss the limiting behaviour of

$$f(x) = \frac{2x}{x^2 - 4} \text{ at } x = \pm 2.$$ (4½)

(b) Evaluate the following :

(i) $\int_0^2 (x^2 + 1)\,dx$

(ii) $\int \sin(ax + b)\,dx \quad (a \neq 0)$ (4)

(c) If $y = A\cos(\ln x) + B\sin(\ln x)$

Show that :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

where A and B are constant. (4)

SECTION II

4. (a) If $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 0 \\ 1 & 4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 3 \\ 1 & 3 & 1 \end{bmatrix}$

Find AB and BA. Is AB = BA ? (4½)

(b) If $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 & 6 \\ 0 & 1 & 5 \\ 3 & 2 & 1 \end{bmatrix}$

Verify that $(A - B)^T = A^T - B^T$ where T stands for transpose. (4)

(c) Verify that $y = A\cos kt + B\sin kt$ is a Solution of the differential equation $\frac{d^2y}{dt^2} + k^2y = 0$ where A, B, k are constants. (4)

5. (a) If $z = y \log x$

then find $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}$. (4½)

(b) For $u = e^{ax} \sin by$,

verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. (4)

(c) Show that $z = \log\sqrt{x^2 + y^2}$

satisfies Laplace's equation

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$. (4)

6. (a) When sugar is dissolved in water, the amount A that remains undissolved after t minutes satisfies the differential equation $\frac{dA}{dt} = -kA\ (k > 0)$. Let A_0 be the value of A for $t = 0$. Find $A = A(t)$. (4)

(b) If the distance S covered by a particle in t seconds is given by $S = \sqrt{t}$, then show that the acceleration of the particle is negative and proportional to the cube of its velocity. (4½)

(c) When a bactericide was added to a nutrient broth, the population 'b' of bacteria at time t(hours) is given by[1]

$$b = 10^6 + 10^5 t - 10^3 t^2.$$

Find the growth rate of population at $t = 40$. (4)

SECTION III

7. (a) Compute the mean and standard deviation of the following distribution of marks in a class of 50 students :

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	8	15	16	6

(6)

(b) Define conditional probability. Dealing with MN blood groups, an individual has either an M antigen or an N antigen or both of them. In a certain population the relative frequencies are : antigen M alone 42%, antigen N alone 33% and antigen M and N combined 25%. A randomly selected individual is found to have an antigen M. What is the probability that this individual also has antigen N ? (6½)

8. (a) The probability that an evening college student will graduate is 0.4. Using Binomial probability distribution determine the probability that out of 5 students (i) none (ii) one (iii) at least one and (iv) all will graduate. (6)

(b) A random variable X has the following probability function :

Values of X, x	−2	−1	0	1	2	3
p(x)	0.1	k	0.2	2k	0.3	k

Find the value of k, and calculate mean and variance. (6½)

9. (a) The mean of a random sample of 100 individuals from a population is 64 inches. The standard deviation of the sample is 4.0 inches. Would it be unreasonable to suppose that the mean of the population is 66 inches ? (Level of significance being 1%). (6)

(b) For 50 students of a class the regression equation of marks in Physics (X) on the marks in Mathematics (Y) is $3Y - 5X + 180 = 0$. The mean marks in Mathematics is 44 and variance of marks in Physics is $\frac{9}{16}$th of the variance of marks in Mathematics. Find mean marks in Physics and the coefficient of correlation between marks in two subjects. (6½)

(200)****