[This question paper contains 4 printed pages.]

2520   *Your Roll No.* ....

MATHEMATICAL SCIENCES (Statistics)

Paper I – Statistical Methods – I

Time : 3 Hours   *Maximum Marks : 38*

(Write your Roll No. on the top immediately
. on receipt of this question paper.)

Attempt **Six** *questions in all, selecting at*
least **two** *questions from each Section.*
Question No. **1** *is compulsory.*

1. (a) The mean of a set of 7 observations is 10 and the
        mean of the other set of 3 observations is 5. Find ·
        the mean of the combined set.

    (b) The median and standard deviation of a distribution
        are 10 and 5 respectively. If each observation is
        increased by 3, find the new median and the new
        standard deviation.

    (c) Determine the parameters of Binomial distribution
        for which mean is 4 and variance is 3.

    (d) The coefficient of rank correlation between marks
        in Statistics and marks in Mathematics obtained

by a certain group of students is 0.8. If the sum
of the squares of the difference in ranks is given
to be 33, find the number of students in the group.

(2,2,2,2)

SECTION A

2. (a) Show that for any discrete distribution, the sum of
squares of the deviations is minimum when taken
about mean.

    (b) Explain the method of constructing Histogram and
Ogive. Also, explain the graphical method of
locating the. values of median and mode.   (3,3)

3. (a) What do you understand by Skewness ? How is it
measured ? Distinguish clearly, by giving figures,
between positive and negative skewness.

    (b) Find the mean deviation from the mean and
standard deviation of a, $a + d$, $a + 2d$, - - -, $a + 2nd$
and verify that the latter is greater than the former.

(3,3)

4. (a) Explain the principle of least squares and describe
its applications in fitting a curve of the form
$Y = ae^{bx}$.

    (b) If X and Y are random variables with correlation
coefficient $r(x, y) = 0.4$, find (i) $r(3x - 2, -2y + 3)$
(ii) $r(3x, 5y)$.   (3,3)

5. (a) Out of the two lines of regression given by $X + 2Y - 5 = 0$ and $2X + 3Y - 8 = 0$, which one is the regression line of X on Y ? Also, find regression coefficients and the correlation coefficient.

(b) Explain 'partial correlation' and 'multiple correlation'. Give expressions for (i) $R_{1.23}$ (ii) $r_{12.3}$ (iii) $\sigma^2_{1.23}$. (3,3)

SECTION B

6. (a) Obtain Poisson distribution as a limiting case of Binomial distribution.

(b) Show that in a Poisson distribution with unit mean, mean deviation about mean is (2/e) times the standard deviation. (3,3)

7. (a) Define negative binomial distribution. Obtain its moment generating function. Hence find its mean and variance.

(b) Find the mode of $X \sim N(\mu, \sigma^2)$. (3,3)

8. (a) If $X \sim$ Gamma (λ), find the limiting distribution of X as $\lambda \to \infty$.

(b) Let $X \sim N(0, 1)$ and $Y \sim N(0, 1)$ be independent random variables. Find the distribution of X/Y and identify it.   (3,3)

9. (a) What do you mean by independence of attributes ? Give a criterion of independence for attributes A and B in terms of ultimate frequencies.

(b) Define beta distribution of first kind with parameters (m, n) and find its mean.   (3,3)

10. (a) If X has an exponential distribution, show that :

$$P(Y \leq x \mid X \geq a) = P(X \leq x) \quad \forall \, x, \; a \geq 0,$$

where $Y = X - a$.

(b) Define Bivariate Normal distribution.

If $(X, Y) \sim B \cup N(0, 0, 1, 1, \rho)$,

find the marginal density function $f_x(x)$ of X.

(3,3)