[This question paper contains 4 printed pages.]

2521 *Your Roll No.* ....

B.Sc. (G)/I **A**

MATHEMATICAL SCIENCES (STATISTICS)

Paper II – Probability

Time : 3 hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** *questions in all.*
Question No. **1** *is compulsory and choose* **five** *from the remaining questions.*

1. (a) For two independent events A and B, the probability that both occur is $\frac{1}{8}$ and probability that neither of them occurs is $\frac{3}{8}$. Find the probability of occurrence of A.

   (b) If the m.g.f. of a random variable X is $(2 - e^t)^{-1}$, find $E(X + 1)$ and $Var(X + 1)$.

   (c) Prove or disprove the following :

   (i) If $P(A) = 0$, then $P(A \cap B) = 0$

   (ii) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, then $A \cap B = \phi$

P.T.O.

(d) If f_1 and f_2 are the p.d.f's and $Q_1 + Q_2 = 1$, check if $g(x) = Q_1 f_1(x) + Q_2 f_2(x)$ is a p.d.f.

(2,2,2,2)

2. (a) Define conditional probability. State whether conditional probability satisfies the axioms of probability.

(b) Find the probability that, in a random arrangement of letters of word UNIVERSITY, the two I's do not come together. (3,3)

3. (a) Four identical marbles marked 1, 2, 3 and 123 respectively are put in a bag and one is drawn at random. Let $A_i (i = 1, 2, 3)$ denote the event that the number i appears on the drawn marble. Check whether A_1, A_2 and A_3 are independent.

(b) In a group of 20,000 men and 10,000 women, 6% of the men and 3% of the women have certain affliction. What is the probability that an afflicted member of a group is a man? (3,3)

4. (a) Given that $f(x) = K\left(\frac{1}{2}\right)^x$, $x = 0, 1, 2, 3, 4, 5, 6$ is a p.m.f. of a random variable X, find K, the corresponding c.d.f. F(x) and E(X).

(b) A continuous random variable X has a p.d.f. $f(x) = 3x^2$, $0 \le x \le 1$. Find a and b such that $P(X \lesseqgtr a) = P(X > a)$ and $P(X > b) = 0.05$. (3,3)

5. (a) State and prove multiplication theorem of expectation.

(b) Let X be a random variable with c.d.f. $F(x) = x$, $0 \le x \le 1$.

Determine the distribution function $F_Y(y)$ of the random variable $Y = \sqrt{X}$ and hence find E(Y).

(3,3)

6. (a) Two ideal dice are thrown. Let X_1 be the score on the first dice and X_2, the score on the second dice. Let Y denote the maximum of X_1 and X_2. Write the joint distribution of Y and X_1. Find the mean and variance of Y.

(b) A player tosses 3 fair coins. He wins Rs. 8 if 3 heads occur, Rs. 3 if 2 heads occur and Re. 1 if one head occurs. If the game is to be fair, how much should he lose, if no heads occur?

(3½,2½)

7. (a) A random variable X has p.d.f. $f(x) = e^{-x}$ for $x \ge 0$. Show that Chebyshev's inequality gives $P(|X-1| > 2) < \frac{1}{4}$ and the actual probability is e^{-3}.

(b) Obtain m.g.f. of the random variable X having p.d.f.

$$f(x) = \begin{cases} x \ ; & 0 \le x < 1 \\ 2-x \ ; & 1 \le x < 2 \end{cases}$$ (3½,2½)

8. (a) State the Weak law of large numbers. Does there exist a variate X for which

$P[\mu_x - 2\sigma \le X \le \mu_x + 2\sigma] = 0.6$?

(b) Let X_n be a sequence of mutually independent random variables such that $X_n = \pm 1$ with probability $\frac{1 - 2^{-n}}{2}$ and $X_n = \pm 2^{-n}$ with probability 2^{-n-1}. Examine whether the weak law of large numbers can be applied to the sequence $\{X_n\}$. (3,3)