[This question paper contains 4 printed pages.]

2507 *Your Roll No.* ....

B.Sc. (G)/I **A**

MATHEMATICS – Paper I

(Algebra)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt any **two** *parts from each question.*

1. (a) Prove that the set of all non-singular matrices of order n with real entries is a group with respect to matrix multiplication.

   (b) Define the order of an element of a group. Prove that for $a \in G$

   (i) $0(a) = 0(x\,ax^{-1})$ for all $x \in G$

   (ii) If $0(a) = n$ and $a^m = e$ then n/m.

   (c) Define a normal subgroup of a group. Prove that intersection of two normal subgroups is a normal subgroup. (5,5,5)

2. (a) Define an integral domain and a field. Prove that a field is an integral domain. Is the converse true ? Justify your answer.

(b) Define an ideal and a subring of a ring. Show that the set

$$\left\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} : a, b \in I\right\}$$

is a right ideal of the ring R of 2×2 matrices over integers. Is A a left ideal of R ?

(c) Prove that a ring R is commutative iff $(a+b)^2 = a^2 + 2ab + b^2$ for all $a, b \in R$.

(4½,4½,4½)

3. (a) Define a vector space. Give an example of a vector space.

(b) Express vector $V=(3, 1, -4)$ as a linear combinition of vectors $V_1=(1, 1, 1)$, $V_2=(0, 1, 1)$, $V_3=(0, 0, 1)$. Is the set $S=\{V, V_1, V_2, V_3\}$ linearly dependent or independent ? Justify your answer.

(c) Let V be an n-dimensional vector space (ie dim V=n) then show that any set of n vectors which is linearly independent is a basis of V.

(4½,4½,4½)

4. (a) Define rank of a matrix. Show that interchaning any two rows of a matrix does not alter its rank.

(b) Find without actually solving, whether the following system possesses a non-trivial solution.

$$x - y + z = 0$$
$$-3x + y - 4z = 0$$
$$7x - 3y - 9z = 0$$
$$4x - 2y + 5z = 0$$

(c) Verify that the matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

satisfies its characteristic equation. It is true for every square matrix ? Hence find its inverse.

(4½,4½,4½)

5. (a) Express cos 7θ and sin 7θ in terms of powers of sinθ and cosθ.

(b) Sum to n terms the series

$\cos\alpha + \cos(\alpha-\beta) + \cos(\alpha-2\beta) + \cdots$.

(c) Using De Moivre's theorem, solve the equation $x^9 + x^5 + x^4 + 1 = 0$. (4½,4½,4½)

6. (a) Solve the equation

$3x^3 + 14x^2 - 28x - 24 = 0$,

the roots being in G.P.

(b) Find a necessary condition for the roots of equation $ax^3 + bx^2 + cx + d = 0$ to be in A.P.

(c) Form the cubic whose roots are the values of α, β, γ given by relations

$$\alpha + \beta + \gamma = 1$$
$$\alpha^2 + \beta^2 + \gamma^2 = 2$$
$$\alpha^3 + \beta^3 + \gamma^3 = 3$$

Find the value of $\alpha^4 + \beta^4 + \gamma^4$. (4½,4½,4½)