[This question paper contains 4 printed pages.]

2508 *Your Roll No.* ....

B.Sc. (G)/I **A**

MATHEMATICS – Paper II

(Calculus)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt any **two** *parts from each question.*

1. (a) Examine if $\lim_{x\to 0} f(x)$ exists when

$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x^2}}}{1 - e^{\frac{1}{x^2}}} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$ (4½)

(b) Let f be a continuous function defined on an internal I, then prove that $|f|$ is continuous on I. Is the converse true ? Give reasons in support of your answer. (4½)

(c) Discuss the derivability of the function :

$$f(x) = \begin{cases} 2x - 3 & \text{when } 0 \le x \le 2 \\ x^2 - 3 & \text{when } 2 < x \le 4 \end{cases}$$

at the point $x = 2$ and at $x = 4$. (4½)

P.T.O.

2. (a) If $Z = \sec^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$,

prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\cot z$$ (4½)

(b) Find the n^{th} derivative of

$$y = e^x \sin^4 x$$ (4½)

(c) If $y = (\sin^{-1} x)^2$,

show that

$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$,

for each positive integer n where y_n denotes n^{th} ordered derivative of y with respect to x. (4½)

3. (a) If the tangent at any point on the curve $x^3 + y^3 = 2a^3$ cuts off lengths p and q on the coordinate axes. Prove that

$$p^{-\frac{3}{2}} + q^{-\frac{3}{2}} = 2^{-\frac{1}{2}} a^{-\frac{3}{2}}$$ (4½)

(b) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents ? If so, where ? (4½)

(c) Show that the tangent and the normal at any point of the curve

$$x = ae^{\theta}(\sin\theta - \cos\theta),\ y = ae^{\theta}(\sin\theta + \cos\theta)$$

are equidistant from the origin. (4½)

4. (a) Find all the asymptotes of the curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$

(4½)

(b) Determine the position and nature of the double points on the curve

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$$ (4½)

(c) Trace the curve

$$y(1 + x^2) = x$$ (4½)

5. (a) Evaluate :

(i) $$\int \frac{dx}{(1+x)\sqrt{x^2-1}}$$ (2)

(ii) $$\int \frac{3\cos x + 4\sin x}{4\cos x + 5\sin x}\,dx$$ (2½)

(b) Find the reduction formula for

$$\int \left(a^2 + x^2\right)^{\frac{n}{2}} dx,$$

where n is a positive integer. (4½)

(c) Evaluate :

(i) $$\int_0^{\frac{\pi}{2}} \log \sin x\,dx$$ (2)

(ii) $\int_{-1}^{2} |2x-3|\, dx$ (2½)

6. (a) Find the area of the region bounded by the curve $y = 4 - x^2,\ 0 \le x \le 3$ and the x-axis. (5)

(b) Find the volume of the solid of revolution formed when the arc of the parabola $y^2 = 4ax$ between the ordinates $x = 0$ and $x = a$ is revolved about its axis. (5)

(c) Find the whole length of the curve :

$x = a\cos^3\theta, \quad y = a\sin^3\theta$ (5)