This question paper contains 5 printed pages.]

5002 *Your Roll No.* .....

B.Sc. (G) / I **B**

MATHEMATICAL SCIENCES

(Operational Research)

Paper II – Optimization-I

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Answer any **five** questions.*

1. (a) Discuss the importance of operational research in decision making. 5

(b) Define the following terms : 6

(i) Basic feasible solution

(ii) A Hyperplane

(iii) Linearly independent set of vectors

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2. (a) Define a convex set and examine the convexity of the set : 4

$S = \left\{(x_1, x_2) : x_1^2 + x_2^2 \leq 1\right\}$.

(b) A firm plans to purchase at least 200 quintals of scrap containing high quality metal X and low quality metal Y. It decides that the scrap to be purchased must contain at least 100 quintals of X metal and not more than 35 quintals of metal Y. The firm can purchase the scrap from two suppliers (A and B) in unlimited quantities. The percentage of X and Y metals in terms of weights in the scrap supplied by A and B is given below :

Metal	Supplier A	Supplier B
X	25%	75%
Y	10%	20%

The price of A's scrap is Rs. 200/- per quintal and that of B is Rs. 400/- per quintal. The firm wants to determine the quantities that it should buy from the two suppliers so that total cost is minimised. Solve it by graphical method. 7

3. Write the different possible situations at the end of phase-I, in the two phase method for solving a given linear programming problem (lpp). 2+9

Use two phase method to solve the following LPP :

Minimize $Z = 600\, x_1 + 400\, x_2$

subject to the constraints

$$3x_1 + 3x_2 \geq 40$$

$$3x_1 + x_2 \geq 40$$

$$2x_1 + 5x_2 \geq 44$$

$$x_1, x_2 \geq 0$$

4. (a) State and prove the complementary slackness theorem in linear programming problem. 6

(b) Write the dual of the following LPP

Min $Z = 5x_1 + 2x_2 + x_3$

subject to the constraints

$x_1 + 2x_2 + x_3 = 15$

$-x_1 + 5x_2 \leq 18$

$4x_2 + 7x_2 \leq 20$

$x_1,\ x_3 \geq 0,\ x_2$ is unrestricted. 5

5. Solve the following Integer linear programming problem :

Max $Z = x_1 + 4x_2$

subject to the constraints :

$2x_1 + 4x_2 \leq 7$

$5x_1 + 3x_2 \leq 15$

$x_1,\ x_2 \geq 0$ and are integers. 11

6. (a) Give a brief outline of the Dual Simplex Method. 5

(b) Solve the following set of linear equations by the simplex method. 6

$$8x_1 + 2x_2 = 15$$

$$5x_1 + 3x_2 = 19$$

7. (a) Solve the assignment problems whose cost matrix is given below : 5

	A	B	C	D
I	1	4	6	3
II	9	7	10	1
III	4	5	11	7
IV	8	7	2	5

(b) Solve the following transportation problem to find the minimum transportation cost : 6

Source	Destination					Available
	D_1	D_2	D_3	D_4	D_5	
S_1	4	7	3	8	2	4
S_2	1	4	7	3	8	7
S_3	7	2	1	5	5	9
S_4	4	3	2	1	5	2
Required	8	3	7	2	2	

8. (a) Discuss degeneracy and method of solving degeneracy in linear programming problems. 4

(b) Explain the term "sensitivity analysis". Consider the following table which present an optimal solution to some linear programming problem.

		Cj	2	4	1	3	2	0	0	0
B	C_B	X_B	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8
y_1	2	3	1	0	0	–1	0	0.5	0.2	–1
y_2	4	1	0	1	0	2	1	–1	0	0.5
y_3	1	7	0	0	1	–1	–2	5	–0.3	2
z = 17		Δ_J	0	0	0	2	0	2	0.1	2

If the additional constraint

$$x_1 + 4x_2 - 2x_3 + 2x_4 - 4x_5 \leq 5$$

were annexed to the system, would there be any change in the optimal solution ? Justify your answer. 7