[This question paper contains 4 printed pages.]

8042

Your Roll No. ....

B.Sc. (G)/I **JS**

MATHEMATICAL SCIENCES (STATISTICS)

Paper II – Probability

Time : 3 hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** *questions in all.*
Question No. 1 *is compulsory and select* **five** *from the remaining questions.*

1. (a) State which of the following statements are true or false. In case of a false statement, give the correct statement :

   (i) Arithmetic mean $\leq$ median $\leq$ mode

   (ii) Kurtosis is equal to one

   (iii) $P(A \cap B) \geq P(A)$

   (iv) If C is any constant then $Var(C) = C$

   (b) For the distribution given below :

X :	–3	–2	–1	0	1	2	3
f(x) :	4	5	7	10	7	5	4

   Check whether it is symmetrical or not.

(c) State the relationship between the first four central moments and cumulants.

(d) The c.d.f. of a random variable X is :

$$F(x) = \begin{cases} 1-e^{-2x}; & x \geq 0 \\ 0 \quad , & \text{otherwise.} \end{cases}$$

Find p.d.f. of X.

(e) If A and B are two mutually exclusive events, show that

$$P(A|A \cup B) = \frac{P(A)}{P(A)+P(B)}; \; P(A \cup B) \neq 0$$

(2,2,1½,1,1½)

2. (a) An event A is known to be independent of the events B, $B \cup C$ and $B \cap C$. Show that it is also independent of C.

(b) An urn contains 4 tickets numbered 1, 2, 3, 4 and another contains 6 tickets numbered 2, 4, 6, 7, 8, 9. If one of the two urns is chosen at random and a ticket is drawn at random from the chosen urn, then find the probability that the ticket drawn bears the number 2 or 4. (3,3)

3. (a) With the usual notations, prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

(b) The sample space consists of integers from 1 to 2n which are assigned probabilities proportional to their logarithms. Find the probabilities. Also find the probability of integer 2 given that an even integer occurs. (3,3)

4. (a) Let p(x) be a p.m.f. of a random variable X which assume the values x_1, x_2, x_3, x_4 such that $2p(x_1) = 3p(x_2) = p(x_3) = 5p(x_4)$. Find the probability distribution and cumulative probability distribution of X. Find the largest x such that $F(x) < \frac{1}{2}$.

(b) Find mean, median and mode for the distribution $f(x) = \sin x;\ 0 \le x \le \frac{\pi}{2}$. (3,3)

5. (a) State and prove multiplication theorem of expectation.

(b) A coin is tossed until a tail appears. Find the expectation of number of tosses. (3,3)

6. (a) Given $f(x, y) = x\, e^{-x(y+1)};\ x \ge 0,\ y \ge 0$.

Find (i) marginal p.d.f. of X (ii) marginal p.d.f. of Y (iii) conditional distribution of Y given X.

(b) A random variable X has p.d.f. $f(x) = 1;\ 0 < x < 1$, find the p.d.f. of $\frac{1}{X}$. (3½,2½)

7. (a) For the given p.m.f. $f(x) = \frac{e^{-1}}{x!}$, $x = 0, 1, 2, ----$, find moment generating function and hence mean and variance.

(b) Use Chebychev's inequality to determine how many times a fair coin must be tossed in order that the probability will be at least 0.90 that the ratio of the observed number of heads to the number of tosses will lie between 0.4 and 0.6. (3½,2½)

8. (a) State the Lindeberg Levy central limit theorem. Also give the importance of central limit theorem.

(b) $\{X_K\}$ is a sequence of independent random variables each taking the values $-1, 0, 1$. Given that $P(X_K = 1) = \frac{1}{K} = P(X_K = -1)$, $P(X_K = 0) = 1 - \frac{2}{K}$. Examine if weak law of large numbers holds for this sequence. (2½,3½)