5001　　　　　　　　　*Your Roll No. ....*

B.Sc. (G) / I　　　　**B**
MATHEMATICAL SCIENCES
Paper I – Essentials for Operational Research (Statistics)

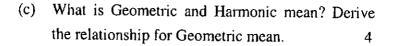*Time : 3 Hours*　　　　　　　　*Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any five questions. Calculators are allowed, provide log tables and normal curve tables.

1. (a) Show that for any distribution, the sum of the deviations from arithmetic mean is always zero.　3

   (b) The daily expenditure of 100 families is given as under

Expenditure	0-10	10-20	20-30	30-40	40-50
Number of families	14	?	27	?	15

   The median and mode for the distribution are Rs. 25 and Rs. 24 respectively. Calculate the missing frequencies.　4

(c) What is Geometric and Harmonic mean? Derive the relationship for Geometric mean. **4**

2. (a) Show that $s^2 = \sigma^2 + d^2$ **4**

   where s = Root mean square deviation

   σ^2 = variance

   $d = \bar{x} - a$

   (b) Calculate the first four moments about the mean of the following distribution, also calculate $\beta_1 + \beta_2$. **7**

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0
f	5	38	65	92	70	40	10

3. (a) State and prove Baye's theorem. **5**

   (b) Show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **3**

   (c) Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. Show that the chance that exactly two of them will be children is $\dfrac{10}{21}$. **3**

4. (a) Three urens of the same appearance have the following proportion of balls.

   First uren   2 black   1 white

   Second uren   1 black   2 white

   Third uren   2 black   2 white

One of the uren is selected and one ball is drawn. It turns out to be white. What is the probability of drawing a white ball again, the first one not having been rebutned. 6

(b) Suppose that X is a Random variable with $E(X) = 10$ and $Var(X) = 25$. Find a and b such that $y = ax - b$ has mean zero and variance one. 5

5. (a) Show that the mean of the binomial distribution is np, where n is the number of trials and p is the probability of success. 3

(b) A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and proportion of days on which some demand is refused $(e^{1.5} = 0.2231)$ 5

(c) Find the mean of the continuous distribution defined by $f(x) = \sin x$ $0 \le x \le \pi/2$. 3

6. (a) Give 5 properties of a normal curve. 3

(b) A variate x is normally distributed with mean 50 and standard deviation 10, find the area under the normal curve between the values of the variate 4

(i) 30 and 40

(ii) 40 and 65

(c) The life of electronic tubes of a certain type may be assumed to be normally distributed with mean 155 hours and standard deviation 19 hours. What is the probability that the life of a randomly chosen tube lies between 136 hours and 174 hours.

4

7. (a) Show that the correlation coefficient between two variables lies between −1 and 1 or $-1 \leq \gamma \leq 1$ or $r^2 \leq 1$.

6

(b) Fit a trend equation of the form $y = a + bx + cx^2$ to the following data and calculate the trend value for 2005.

5

Year	1998	1999	2000	2001	2002	2003	2004
Total sales of a firm	12	14	12	26	42	40	50

8. (a) What is Monte Carlo simulation ? Describe the idea of random sampling is simulation.

5

(b) Define the following :

6

(i) Markov chains

(ii) Equilibrium of steady state

(iii) Transition probabilities