[This question paper contains 4 printed pages.]

5008

Your Roll No............

B.Sc. (G) / I **B**

MATHEMATICS – Paper II

(Calculus)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt any **two** *parts from each question.*

1. (a) If a function f defined on an internal I is derivable at a point $x = c \in I$, then prove that f is continuous at $x = c$. Is the converse true? Give reasons in support of your answer. (4½)

(b) Find a non-zero value for the constant K that makes the function :

$$f(x) = \begin{cases} \dfrac{\tan Kx}{x} & \text{when } x < 0 \\ 3x + 2K^2 & \text{when } x \geq 0 \end{cases}$$

continuous at the point $x = 0$. (4½)

(c) Examine if $\lim_{x \to 0} f(x)$ exists, when

$$f(x) = \frac{(x^2 + 2)|x|}{x}$$ (4½)

2. (a) If $y = e^{m\sin^{-1}x}$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$$

where y_n denotes n^{th} ordered derivative of y with respect to x. (4½)

(b) State Euler's theorem for a homogeneous function of x and y.

If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$$ (4½)

(c) Find the n^{th} derivative of

$$y = e^{ax}\cos(bx+c)$$ (4½)

3. (a) Find the angle of intersection of the curves $y^2 = 2x$ and $x^2 + y^2 = 8$ at a point of intersection. (4½)

(b) Show that the curves

$$r = a(1+\cos\theta) \text{ and}$$

$$r = b(1-\cos\theta)$$

intersect each other orthogonally. (4½)

(c) If p_1 and p_2 be the length of perpendicular from the origin on the tangent and normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ respectively, then prove that $4p_1^2 + p_2^2 = a^2$. (4½)

4. (a) Find all the asymptotes of the curve :

$$x^3 + x^2y - xy^2 - y^3 - 2x^2 + 2y^2 + x + y + 1 = 0$$

(4½)

(b) Determine the position and nature of the double points on the curve :

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$$ (4½)

(c) Trace the curve :

$$8a^2y^2 = x^2(a^2 - x^2), \ a > 0$$ (4½)

5. (a) Evaluate :

(i) $\int \frac{dx}{\cos(x-a)\sin(x-b)}$ (2)

(ii) $\int \frac{dx}{\sin^{\frac{3}{2}} x \cos^{\frac{5}{2}} x}$ (2½)

(b) Evaluate :

(i) $\int_0^{\frac{\pi}{2}} \sin 2x \log \tan x \, dx$ (2)

(ii) $\int_0^{\frac{\pi}{2}} \log(\tan x + \cot x) \, dx$ (2½)

(c) If $u_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, then prove that

$$u_n + u_{n-2} = \frac{1}{n-1}, \ n > 1$$

and hence deduce the value of u_5. (4½)

6. (a) Find the area enclosed by the curve $r = a\cos 2\theta$ and the radius vectors $\theta = 0$ and $\theta = \frac{\pi}{2}$. (5)

(b) Show that the length of the arc of the curve $x = e^{\theta}\sin\theta$, $y = e^{\theta}\cos\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$ is $\sqrt{2}\left(e^{\frac{\pi}{2}} - 1\right)$. (5)

(c) Find the volume of the solid generated by loop of the curve $y^2(a + x) = x^2(3a - x)$ as it revolves about x-axis. (5)