8181 *Your Roll No.............*

B.Sc. (G) / I B

MATHEMATICAL SCIENCES (Operational Research)

Paper II - Optimization – I

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **five** *questions.*

1. (a) Discuss applications of Operational Research and advantages of Operational Research approach in decision making. (5)

   (b) A company manufactures two kinds of machines, each requiring a different manufacturing technique. The deluxe machine requires 18 hours of labour, 8 hours of testing and yields a profit of Rs. 400. The standard machine requires 3 hours of labour, 4 hours of testing and yields a profit of Rs. 200. There are 800 hours of labour and 600 hours of testing available each month. A marketing forecast has shown that the monthly demand for the standard machine is to be more than 150. The management wants to know the number of each model to be produced monthly that would maximize total profit. Formulate a linear programming problem and solve it graphically. (6)

2. (a) Define a convex set. Is the intersection of any finite number of convex sets necessarily a convex set ? Justify.   (6)

(b) Find all the basic feasible solutions of the equations :

$$x_1 - 2x_2 + x_3 = 4$$
$$3x_1 - x_2 - 5x_3 = 5 \qquad (5)$$

3. (a) Define and explain the following terms :

(i) Optimum solution

(ii) Feasible solution

(iii) Degenerate basic feasible solution   (3)

(b) Use simplex method to solve the following LP problem.

Maximize $Z = 3x_1 - 5x_2 + 4x_3$

subject to the constraints,

$$2x_1 - 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$
and $\quad x_1, x_2, x_3 \geq 0$ (8)

4. Explain the use of artificial variables in LP. Use two-phase simplex method to solve the following LP problem.

Minimize $Z = x_1 + x_2$
Subject to the constraints,
$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
and $x_1, x_2 \geq 0$ (11)

5. (a) What is integer linear programming ? How does the optimal solution of an integer programming problem compare with that of the linear programming problem ?   (4)

   (b) Solve the following integer programming problem using Gomory's cutting plane algorithm.
       Maximize $Z = x_1 + x_2$
       Subject to the constraints
       $$3x_1 + 2x_2 < 5$$
       $$x_2 \leq 2$$
       and $x_1, x_2 \geq 0$ and are integers   (7)

6. (a) What is duality ? What is the significance of dual variables in an LP model ?   (3)

   (b) Write the dual of the following linear programming problem.
       Min $Z = 2x_1 - 3x_2 + 4x_3$
       Subject to
       $$2x_1 + 3x_2 + 5x_3 \geq 2$$
       $$3x_1 + x_2 + 7x_3 = 3$$
       $$x_1 + 4x_2 + 6x_3 \leq 5$$
       $x_1, x_2 \geq 0$ and x_3 is unrestricted   (3)

(c) Explain the term "sensitivity analysts". Discuss the effect of discrete changes in the requirement vector (b). (5)

7. (a) Goods have to be transported from sources S_1, S_2 and S_3 to destinations D_1, D_2 and D_3. The transportation cost per unit, capacities of the sources, and the requirements of the destinations are given in the following table. Find the optimal distribution so as to minimize the cost of transportation.

Destination / Source	D.	D_2	D_3	Supply
S.	8	5	6	120
S_2	15	10	12	80
S_3	3	9	10	80
Demand	150	80	50	

(6)

(b) A company has four machines that are to be used for three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table.

		W	X	Y	Z
	A	18	24	28	32
Jobs	B	8	13	17	18
	C	10	15	19	22

(heading: Machines)

What are the job-assignment pairs that shall minimize the cost ? (5)