[This question paper contains 4 printed pages.]

121 *Your Roll No.* ....

B.Sc. (G) / I **C**

MATHEMATICAL SCIENCES (STATISTICS)

Paper II – Probability

Time : 3 hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** *questions in all.*
Question No. **1** *is compulsory and select* **five** *from the remaining questions.*

1. (a) A man has tossed two fair dice. Find the conditional probability that he has tossed two sixes given that he has tossed at least one six.

(b) If $A \cap B = O$ then show that $P(A) \leq P(\bar{B})$.

(c) If a random variable takes values 0 & 1 only, show that all its moments about origin are equal.

(d) (i) State the relation between first four central moments and moments about origin.

(ii) The c.d.f. of a random variable X is

$$F(x) = \frac{x-a}{b-a};\ a < x < b$$

Find its p.d.f. (2,2,2,2)

2. (a) Suite the axioms of probability and explain their frequency interpretations.

(b) Suppose that events $A_1, A_2, \ldots, A_n$ are independent and

$$P\left(A_i = \frac{1}{i+1}, i = 1, 2, \ldots n\right).$$

Find the probability that at least one of them occurs. (3,3)

3. (a) In a group of 20,000 men and 10,000 women. 6% of the men and 3% of the women have certain affliction. What is the probability that an afflicted member of a group is a man ?

(b) Three newspapers A, B and C are published in a city. It is estimated from a survey that 20% read A, 16% read B, 14% read C, 8%, read A and B, 5% read A and C. 4% read B and C and 2% read all the newspapers. What is the probability that a randomly chosen person (i) does not read any paper, (ii) does not read C, (iii) reads A but not B, (iv) reads only one of these papers and (v) reads only two of these newspapers. (3,3)

4. (a) Two ideal dice are thrown. Let X_1 be the score on the first dice and X_2 the score on the second dice. Let Y denote the maximum of X_1 and X_2. Write the joint distribution of Y and X_1. Find the mean and variance of Y.

(b) Calculate the coefficient of variation for the random variable X taking values in (0, b) with the probability law being $P(X \le t) = \frac{t}{b}$. (3.5,2.5)

5. (a) The joint p.d.f. of two random variables X & Y is $f(x,y) = 6x^2y;\ 0 < x < 1,\ 0 < y < 1$. Find

(i) $P\left(0 < X < \frac{3}{4}, \frac{1}{3} < Y < 2\right)$.

(ii) $P(X + Y < 1)$.

(iii) $P(X > Y)$.

(iv) $P(X < 1 \mid Y < 2)$.

(b) The p.d.f. of a random variable X is given by :

$$f(x) = kxe^{-x^2/2},\ x > 0,$$

where k is a constant. Find the median of the distribution. (3,3)

6. (a) Find the expectation of the sum of the number of points on n dice when thrown.

(b) A random variable X has the p.d.f. $f(x) = A(x + 1)$; $-1 < x < 1$. Find A and first four central moments about mean. (3,3)

7. (a) For the given distribution :

$$f(x) = \frac{e^{-1}}{x!}, \; x = 0, 1, 2, \ldots,$$

find m.g.f. cumulant generating function and the first four cumulants.

(b) A random variable X has p.d.f. $f(x) = e^{-x}$ for $x \geq 0$. Show that Chebyshev's inequality gives $P(|X - 1| > 2) < \frac{1}{4}$ and the actual probability is e^{-3}. (3,3)

8. (a) If X_i can have only two value i^{α} and $-i^{\alpha}$ with equal probabilities. show that the law of large numbers can be applied to the independent variables $X_1, X_2, \ldots$ if $\alpha < \frac{1}{2}$.

(b) Is there any relation between CLT and WLLN ? Out of CLT and WLNN, which result is stronger and why ? (3.5,2.5)