[This question paper contains 4 printed pages.]

108 *Your Roll No.* ....

B.Sc. (G) / I **C**

MATHEMATICS – Paper II

(Calculus)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt any **two** *parts from each questions.*
Questions **1** *to* **5** *carry* **9** *marks each.*
Q. **6** *carries* **10** *marks.*

1. (a) Examine if $\lim_{x \to 0} f(x)$ exists when

$$f(x) = \cos\frac{1}{x}$$ (4½)

(b) Examine the function 'f' defined by

$$f(x) = \begin{cases} \dfrac{e^{1/x}}{1 + e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

for continuity at x = 0. (4½)

(c) A function 'f' is defined as follows

$$f(x) = \begin{cases} x & , \quad x < 1 \\ 2 - x & , \quad 1 \le x \le 2 \\ -2 + 3x - x^2, & \quad x > 2 \end{cases}$$

Examine for differentiability at $x = 1$ and $x = 2$. (4½)

2. (a) If $p^2 = a^2\cos^2\theta + b^2\sin^2\theta$, prove that

$$p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$$ (4½)

(b) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that

$$x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0.$$

Where y_n denotes the nth derivative of y with respect to x. (4½)

(c) If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$,

show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u .$$ (4½)

3. (a) Show that the equation of tangent at any point t on the curve whose equation are

$$x = a(t + \sin t); \quad y = a(1 - \cos t)$$

is given by

$$y = (x - at)\tan\frac{t}{2}.$$ (4½)

(b) Find the equation of the normal to the curve $2x^2 - y^2 = 14$ which is parallel to $x + 3y = 4$. (4½)

(c) Find the condition that the conics

$$ax^2 + by^2 = 1 \quad \text{and} \quad a'x^2 b'y^2 = 1$$

may cut orthogonally. (4½)

4. (a) Find all the asymptodes of the curve

$$y^3 + x^2y + 2xy^2 - y + 1 = 0$$ (4½)

(b) Trace the curve : $x^3 + y^3 = 3axy$. (4½)

(c) If ρ and ρ' are the radii of the curvature at the extremities of two semi-diameters of an ellipse, then show that

$$\left[\rho^{2/3} + (\rho')^{2/3}\right]a^{2/3}b^{2/3} = \left(a^2 + b^2\right).$$ (4½)

5. (a) Evaluate $\int \frac{dx}{\left(2x^2 + 3\right)\sqrt{3x^2 - 4}}$. (4½)

(b) Show that

$$\int_0^{\pi/2} \log \sin x \, dx = \frac{\pi}{2} \log \frac{1}{2}.$$ (4½)

(c) If $u_n = \int_0^{\pi/2} x^n \sin x \, dx \quad (n > 1)$,

then prove that

$$u_n + n(n-1)u_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$

Deduce the value of u_5. (4½)

6\. (a) Show that the length of the loop of the curve $3ay^2 = x(x-a)^2$ is $\frac{4a}{\sqrt{3}}$. (5)

(b) Find the area of the smaller portion enclosed by the curves

$$x^2 + y^2 = 9, \quad y^2 = 8x$$ (5)

(c) Find the volume of the solid generated by the revolution of the curve

$$(a - x)y^2 = a^2x$$

about its asymptode. (5)