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B.Sc. Prog./I   C

MA-107 (*a*)—MATHEMATICS—I

(For Physical Sciences)

(NC—Admission of 2008 onwards)

Time : 3 Hours   *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* questions from each Section.

Section I

1.   (*a*)   A ship is travelling across a river at the rate of 8 miles

per hour while the river's current is flowing southward

at the rate of 6 miles per hour. If the ship is travelling

eastward, find the resultant velocity of the ship.

Also, sketch the approximate direction of the resultant

velocity.   6

(b)   Show that the vectors $(1, 2, 1)$, $(2, 1, 0)$ and $(1, -1, 2)$

form a basis for $\mathbf{R}^3$.   6

2.   (a)   Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be defined by :

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ -y \end{bmatrix}.$$

Show that T is a linear transformation.   6

(b)   Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation given by :

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} y \\ x \end{bmatrix}.$$

Find the eigenvalues and the eigenvectors of T.   6

3.   (a)   Solve, by elementary row operations, the following

system of equations :   6

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

(b)   Find the rank of the following matrix :   6

$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}.$$

Section II

4.   (a)   Examine the convergence of the sequence :   6

$$\left\{ (-1)^n \cdot \frac{1}{n} \right\}.$$

(b)   Find the nth derivative of $y = \cos^4 x$.   6

(c)   If

$$y = a \cos(\ln x) + b \sin(\ln x),$$

show that :

$$x^2 y_{n+2} + (2n+1) x y_{n+1} + \left(n^2 + 1 \right) y_n = 0.$$ 6

5.   (a)   Sketch the graph of $y = \sqrt{x+3}$.   6

(b)   A culture of bacteria is grown under perfect conditions in a laboratory so that the population of bacteria grows exponentially with time. At the end of the second hour, there are 4000 bacteria. At the end of the sixth hour, there are 64000 bacteria. How many bacteria were present initially ?   6

(c)   Find the Maclaurin's series expansion of $y = e^x$. assuming :

$$\lim_{n \to \infty} R_n(x) = 0.$$ 6

6.   (a)   Draw the level curves at heights $k = 0, 1, -2$ of the function :

$$Z = -2x - 3y + 5.$$ 6

(b)   Find the slope of the surface :

$$Z = \sqrt{1 - \left(4x^2 + 9y^2\right)}$$

along x-axis at :

$$\left(\frac{1}{4}, \frac{1}{9}, \frac{\sqrt{23}}{6}\right).$$ 6

(c)   Verify that the function :

$$W(x, t) = e^{c^2 \pi^2 t} \left(A e^{\pi x} + B e^{-\pi x} \right),$$

where A, B are constants, satisfies the heat equation :

$$W_{xx} = \frac{1}{c^2} W_t.$$ 6

Section III

7.   (a)   In a certain college, 60% of the students engage in

football, 50% of the students in basketball and 30%

of the students in both football and basketball. If a

student is selected at random, what is the probability

that he will :   4

(i)   play football or basketball ?

(ii)   play neither sport ?

(b)   Determine the binomial probability distribution for

which mean = 3, variance = 2. Find the probability of

no success.   3½

8. (*a*) Show that mean and variance of the Poisson distribution

coincide.   3½

(*b*) For a continuous distribution defined by :

$$dF = y_0 x(2 - x)\ dx,\ 0 \le x \le 2,$$

y_0 is constant. Find the mean and variance.   4

9. (*a*) Fit a straight line to the following data taking y as

dependent variable :   3½

X	Y
1	1
3	2
4	4
6	4
8	5
9	7
11	8

(*b*) The life of electronic tubes of a certain type may be assumed to be normally distributed with mean 155 hours and standard deviation 19 hours. What is the probability that the life of a randomly chosen tube is between 136 hours and 174 hours ?

(Given : P(1) = .3413, P(1.5) = .4332) 4