This question paper contains 4+2 printed pages]

Your Roll No. .....

8058

B.Sc. (G)/I **D**

MATHEMATICS—Paper II

(Calculus)

Time : 3 *Hours* *Maximum Marks* : 55

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions.

All questions carry equal marks.

Attempt any *two* parts from each question.

1. (*a*) Use (ε, δ) definition to prove that :

(*i*) $\lim_{x \to 0} x \sin \frac{1}{x} = 0$

(*ii*) $\lim_{x \to \infty} \frac{1}{x^2} = 0$.

P.T.O.

(*b*) Show that the function defined by :

$$f(x) = |x| + |x-1|$$

is continuous at $x = 0$ and $x = 1$ but not differentiable at these points.

(*c*) Prove that if the functions f and g are continuous at a point then $f + g$ is also continuous at that point what about its converse ? Justify your answer.

2. (*a*) Find the nth derivative of :

$$y = \tan \frac{2x}{1-x^2}.$$

(*b*) If

$$y = \log(x + \sqrt{(1+x^2)}),$$

then show that :

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0.$$

(*c*) If

$$u = f(x+ay) + g(x-ay),$$

then prove that :

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 u}{\partial y^2}.$$

3. (*a*) The tangent at any point on the curve

$$x^3 + y^3 = 3axy,$$

cuts off lengths p and q on the co-ordinate axes, show that :

$$p^{-\frac{3}{2}} + q^{-\frac{3}{2}} = \frac{1}{\sqrt{2}}\, a^{-\frac{3}{2}}.$$

(*b*) Show that the sub-tangent at any point of the curve :

$$x^m y^n = a^{m+n}$$

varies as the abscissa.

P.T.O.

(*c*) Show that the curves

$$r^m = a^m \cos m\theta,\ r^m = a^m \sin m\theta,$$

cut each other orthogonally.

4. (*a*) Find the asymptotes of the curve :

$$x^3 + y^3 = 3axy .$$

(*b*) Trace the curve :

$$y(a^2 + x^2) = a^2 x .$$

(*c*) Find the equations of the tangent at (2, 1) to the curve :

$$(x-2)^2 = y(y-1)^2$$

and show that this point is a node.

5. (*a*) Evaluate :

(*i*) $\int \frac{(x+2)}{\sqrt{(x^2+3x+1)}}\,dx$

(*ii*) $\int \frac{dx}{\sin x+\sin 2x}$.

(*b*) Obtain the reduction formula for $\int \sin^n x\,dx$ where n being positive integer and hence evaluate :

$$\int_0^{\frac{\pi}{2}} \sin^6 x\,dx.$$

(*c*) Prove that :

$$\int_0^{\pi} \frac{x\tan x\,dx}{\sec x+\tan x} = \frac{\pi}{2}(\pi-2).$$

6. (*a*) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

(*b*) Find the volume of solid of revolution of the cardioid :

$$r = a(1 - \cos\theta)$$

is revolved about initial line.

(*c*) Rectify the curve :

$$x = a(t + \sin t),\ y = a(1 - \cos t).$$