[This question paper contains 3 printed pages.]

4694 *Your Roll No.* .....

B.Sc. (G)/II **AS**

MATHEMATICS – Paper IV

(Vector Calculus and Differential Equations)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt any **Two** *parts from each question.*

1. (a) Find the direction along which the directional derivative of the function $xy + 2yz + 3xz$ is greatest at the point $(1, 1, 1)$. Also find the greatest directional derivative. (5)

   (b) Prove that $\nabla \times \nabla |\vec{r}|^m = 0$. (5)

   (c) Prove that

   $$\text{grad}(\vec{f} \cdot \vec{g}) = \vec{f} \times \text{curl}\,\vec{g} + \vec{g} \times \text{curl}\,\vec{f} + (\vec{f} \cdot \nabla)\vec{g} + (\vec{g} \cdot \nabla)\vec{f}$$

   (5)

2. Solve the following differential equations :

   (a) $y - x\frac{dy}{dx} = a^2\left(y + \frac{dy}{dx}\right)$ (4.5)

P.T.O.

(b) $\tan x \cos y \, dy + \sin y \, dx + e^{\sin x} \, dx = 0$. (4.5)

(c) $y + px = p^2x^4$. (4.5)

3. (a) Solve :

$(D^2 - 4D + 3)y = e^x \cos 2x + \cos 3x$. (4.5)

(b) Solve : $(D^2 + 1)y = \sec^2 x$. (4.5)

(c) Define Wronskian of the two solutions $y_1(x)$ and $y_2(x)$. Show that $y_1(x) = \sin x$ and $y_2(x) = \sin x - \cos x$ are linearly independent solutions of $y'' + y = 0$. (4.5)

4. Solve the following differential equations :

(a) $\frac{d^2y}{dx^2} - \frac{x}{x-1}\frac{dy}{dx} + \frac{y}{x-1} = x - 1$. (4.5)

(b) $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + n^2y = 0$. (4.5)

(c) $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + \cos^2 x \cdot y = 0$. (4.5)

5. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = 4 \tan 2x$$

by the method of variation of parameters. (4.5)

(b) Solve the differential equation

$(D^3 + 3D^2 + 2D)y = x^2 + 4x + 8$

by the method of undetermined co-efficients. (4.5)

(c) Solve :

$$\frac{d^2y}{dx^2} - \frac{6}{x^2}y = x \log x \qquad (4.5)$$

6. (a) Solve the following simultaneous differential equations

$$\frac{d^2x}{dt^2} - 3x - 4y = 0$$

$$\frac{d^2y}{dt^2} + x + y = 0 \qquad (4.5)$$

(b) Solve :

$$\frac{dx}{y^2+z^2} = \frac{dy}{-xy} = \frac{dz}{-xz} \qquad (4.5)$$

(c) Solve :

$$(x^2z - y^3)dx + 3xy^2\, dy + x^3\, dz = 0 \qquad (4.5)$$