[This question paper contains 4 printed pages.]

2509 *Your Roll No.* ....

B.Sc. (G)/II **A**

MATHEMATICS – Paper III

(Geometry)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **All** *questions, selecting* **two** *parts each from Q. No.* **1, 2, 3, 5, 6** *(***Nine** *marks each) and* **one** *part from Q. No.* **4** *(***10** *marks).*
All *questions carry equal marks.*

1. (a) Prove that the two circles which pass through the points (0, a) and (0, –a) and touch the line $y = mx + c$ will cut orthogonally if $c^2 = a^2(1 + m^2)$.

   (b) Find the limiting points of the system of co-axial circles determined by the circles

   $$x^2 + y^2 + 2x - 6y = 0 \quad \text{and}$$
   $$2x^2 + 2y^2 - 10y + 5 = 0$$

   (c) Find the equation of the circles which cuts the circles

   $$x^2 + y^2 - 4x - 6y + 11 = 0 \quad \text{and}$$
   $$x^2 + y^2 - 10x - 4y + 21 = 0$$

   orthogonally and passes through the point (1, 1).

P.T.O.

2. (a) Find the area of the triangle ABC formed by a focal chord, AB of the parabola $y^2 = 8x$, where A is the point (8, 8) and C is the point of intersection of tangents to the parabola at A and B.

(b) Show that from a point three normals can be drawn to the parabola $y^2 = 4ax$ and sum of the ordinates of feet of these normals is zero.

(c) Show that the locus of the poles of the normal chords of the parabola $y^2 = 4ax$ is

$$(x + 2a)y^2 + 4a^3 = 0$$

3. (a) Find the ratio of areas of an ellipse (of eccentricity $\frac{1}{\sqrt{2}}$), its auxilliary circle and its director circle.

(b) Prove that the locus of midpoints of portion of tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ included between the axes is the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$.

(c) Prove that the locus of poles of normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the curve $\frac{a^6}{x^2} + \frac{b^6}{y^2} = \left(a^2 + b^2\right)^2$.

4. Trace any **one** of the following conics giving essential details

(a) $3x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0$

(b) $25x^2 - 120xy + 144y^2 - 2x - 29y - 1 = 0$.

5. (a) Find the equation of the sphere through the four points (0, 0, 0), (0, 1, –1), (–1, 2, 0), (1, 2, 3).

(b) Obtain the equation of the circle lying on the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 3 = 0$, and having its centre at (2, 3, –4).

(c) Find the equation of the sphere which cuts the sphere

$$x^2 + y^2 + z^2 + 6x - 4y + 4 = 0$$

orthogonally and touches the plane

$$2x + 3y - z + 2 = 0 \text{ at } (-2, 1, 1).$$

6. (a) Find the equation of enveloping cone of the sphere

$$x^2 + y^2 + z^2 + 2x - 2y + 2z - 1 = 0,$$

with its vertex at (2, 4, 3).

(b) Find the points of intersection of the line

$$\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2}$$

with the cone $11x^2 - 5y^2 + z^2 = 0$.

(c) Find the equation of the right circular cylinder whose guiding circle is

$$x^2 + y^2 + z^2 = r^2, \quad x + y + z = 0.$$