[This question paper contains 3 printed pages.]

2510 *Your Roll No.* .....

B.Sc. (G)/II **A**

MATHEMATICS – Paper IV

(Vector Calculus and Differential Equations)

Time : 3 hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt any **two** *parts from each question.*
Each part of each question upto **5** *carries* **4½** *marks but in Question No.***1**, *each part carries* **5** *marks.*

1. (a) Find the directional derivative of the function

$$\phi(x, y, z) = xy + yz + zx$$

in the direction of the vector $i + 2j + 2k$ at the point $(1, 2, 0)$. (5)

(b) Find $\nabla\varphi$, if (i) $\varphi = \dfrac{1}{|\vec{r}|}$

(ii) $\varphi = |\vec{r}|^m$ (5)

(c) Prove that

$$\text{Curl}\left(\vec{f} \times \vec{g}\right) = \vec{f}\ \text{div}\ \vec{g} - \vec{g}\ \text{div}\ \vec{f} + \left(\vec{g} \cdot \nabla\right)\vec{f} - \left(\vec{f} \cdot \nabla\right)\vec{g}$$

(5)

P.T.O.

2. (a) Solve : $x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$. (4.5)

(b) Solve : $(x^2y^3 + xy)dy = dx$. (4.5)

(c) Solve : $xy\,p^2 + p(3x^2 - 2y^2) - 6xy = 0$. (4.5)

3. (a) Solve : $(D^3 + 1)y = \sin 3x - \cos^2\frac{x}{2}$. (4.5)

(b) Solve : $(D^2 + 4)y = \sin 2x$. (4.5)

(c) Show that linearly independent solutions of $y'' - 3y' + 2y = 0$ are e^x and e^{2x}. Find the solution $y(x)$ with the property that $y(0) = 0$, $y'(0) = 1$. (4.5)

4. (a) Solve :

$$x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3. \quad (4.5)$$

(b) Solve :

$$\left(\frac{d^2y}{dx^2} + y\right)\cot x + 2\left(\frac{dy}{dx} + y\tan x\right) = 0. \quad (4.5)$$

(c) Solve :

$$x^4\frac{d^2y}{dx^2} + 2x^3\frac{dy}{dx} + 4y = 0. \quad (4.5)$$

5. (a) Solve :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$$

by the method of variation of parameters. (4.5)

(b) Solve :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x$$

by the method of undetermined co-efficients. (4.5)

(c) Solve :

$$[(1+x)^2D^2 + (1+x)D + 1]y = 4\cos[\log(1+x)].$$ (4.5)

6. (a) Solve the following Simultaneous differential equations :

$$\frac{dx}{dt} + 5x + y = e^t$$

$$\frac{dy}{dt} - x + 3y = e^{2t}$$ (4.5)

(b) Solve :

$$\frac{dx}{x^2} + \frac{dy}{y^2} = \frac{dz}{nxy}.$$ (4.5)

(c) Solve :

$$(x^2 + y^2 + z^2)dx - 2xy\,dy - 2xz\,dz = 0.$$ (4.5)