[This question paper contains 4 printed pages.]

4693 *Your Roll No.* .....

B.Sc. (G)/II **AS**

MATHEMATICS – Paper III

(Geometry)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **all** *questions, selecting* **two** *parts from each of the questions* **1** *to* **5.**

1. (a) Show that two circles

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and

$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ are

orthogonal if

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2 \qquad (9)$$

(b) The Radical Axis of coaxial system of circles is the line $x + y = 0$, if one of the members is the unit circle with centre at the origin, then find its limiting points or points of intersection; which ever are real.

P.T.O.

(c) Find the equation of a circle which is member of a coaxial system determined by the circles

$x^2 + y^2 + 2x = 0$ and

$x^2 + y^2 - 2x - 4y + 1 = 0$

and passing through the point $(1, -1)$.

2. (a) If normal at a point $(at^2, 2at)$ of the parabola $y^2 = 4ax$ meets it again at $(at'^2, 2at')$ prove that

$$t' = -\left(t + \frac{2}{t}\right). \qquad (9)$$

(b) Prove that the locus of the middle points of a system of parallel chords of a hyperbola is a straight line.

(c) Find locus of poles of the tangents to the circle

$x^2 + y^2 = 4a^2$ with respect to the parabola $y^2 = 4ax$.

3. (a) Find the locus of a point from which two perpendicular tangents can be drawn to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. By what name is this curve known ?

(b) Find the locus of midpoints of chords of hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which are tangent to the circle

$x^2 + y^2 = r^2$.

(c) Tangent at the point P(9, 4) on the hyperbola $\frac{x^2}{45} - \frac{y^2}{20} = 1$ meets its asymptotes at points Q and R. Find coordinates of Q and R. (9)

4. (a) Obtain the equation of the sphere circumscribing the tetrahedron whose faces are

$$x = 0,\ y = 0,\ z = 0;\quad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1. \qquad (9)$$

(b) Find the equation of the sphere containing the circle

$x^2 + y^2 + z^2 + 2x - 4y + 6z + 5 = 0;\ x + 2y + 3z - 8 = 0$

as a great circle.

(c) Find the limiting points of the coaxial system of spheres determined by the spheres

$x^2 + y^2 + z^2 + 3y - 3z + 6 = 0$

and $x^2 + y^2 + z^2 + 2x + 4y - 2z + 6 = 0$

5. (a) Find the equation of the cone with vertex at (4, 3, 5) which passes through the circle $x^2 + y^2 + z^2 = 4\quad x + y = 0$. (9)

(b) Find the points of intersection of the line

$\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2}$ with the cone

$11x^2 - 5y^2 + z^2 = 0$.

P.T.O.

(c) Find the equation of the cylinder whose generators intersect the curve

$x^2 + y^2 + z^2 = 9$; $x + y + z - 2 = 0$ and are parallel to the line

$$\frac{x}{2} = \frac{y}{-1} = \frac{z-1}{-2}$$

6. Trace **any one** of the following conics giving essential details

(i) $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$

(ii) $25x^2 - 120xy + 144y^2 - 2x - 29y - 1 = 0$

(10)