

[This question paper contains 4 printed pages.]

5009

Your Roll No.....

B.Sc. (G) / II

B

MATHEMATICS – Paper III

(Geometry)

Time : 3 Hours

Maximum Marks : 55

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*Attempt All questions,
selecting two parts from each question.*

1. (a) Find the equation of the circle cutting each of the circles

$$x^2 + y^2 + 2x + 4y - 10 = 0 \text{ and}$$

$$x^2 + y^2 - 4x + 6y - 3 = 0$$

orthogonally and passing through $(-1, 4)$

- (b) Define radical axis of two circles. Show that it is straight line perpendicular to the line joining their centres.

- (c) Find the equation of a circle which passes through the origin and is a member of coaxial system of circles whose limmiting points are $(1, -1)$ and $(2, 1)$. (9)

P.T.O.

2. (a) Prove that tangents at the extremities of a focal chord of a parabola are perpendicular to each other and they intersect on the directrix of the parabola.

(b) Show that locus of midpoints of focal chords of the parabola $y^2 = 4ax$ is the parabola $y^2 = 2a(x - a)$.

(c) Define conjugate lines with respect to a parabola. Derive the condition that the lines $\ell x + my + n = 0$ and $\ell_1 x + m_1 y + n_1 = 0$ are conjugate with respect to the parabola $y^2 = 4ax$. (9)

3. (a) How many tangents can be drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with a given slope m ? Also find their equations.

(b) Show that from a point four normals can be drawn to an ellipse and that the sum of the eccentric angles of the feet of these normals is an odd multiple of $\pi/2$.

(c) Show that the line $x \cos \alpha + y \sin \alpha = p$ will be tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ if } a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2 \quad (9)$$

4. (a) If the sphere

$$x^2 + y^2 + z^2 + 2u_n + 4y - 6z + K = 0$$

cuts the sphere

$$x^2 + y^2 + z^2 - 8x + 2y + 4z + 5 = 0$$

orthogonally and passes through $(1, 3)$, then find the values of u and K .

(b) Find the centre and radius of the circle

$$x^2 + y^2 + z^2 + 2x - 2y - 4z - 49 = 0;$$

$$x + 2y + 2z + 7 = 0$$

(c) Find the equations of the spheres of the coaxial system

$$x^2 + y^2 + z^2 - 5 + \lambda (2x + y + 3z - 3) = 0$$

which touch the plane

$$3x + 4y = 15 \quad (9)$$

5. (a) Find the equation of the elliptic cone where vertex is at the origin and intersects the ellipse

$$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \text{and} \quad x = a.$$

(b) Find the equations of lines of intersection of the cone $x^2 - 5y^2 + z^2 = 0$ with the plane $x - 3y + z = 0$. Also find the angle between the lines.

(c) Find the equations of the right circular cylinder whose guiding circle is

$$x^2 + y^2 + z^2 = r^2; \quad x + y + z = 0 \quad (9)$$

6. Trace any one of the following conics giving essential details :

(i) $9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$

(ii) $3x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0$

(10)