[This question paper contains 3 printed pages.]

5010 *Your Roll No.....*

B.Sc. (G) / II **B**

MATHEMATICS – Paper IV

(Vector Calculus and Differential Equations)

Time : 3 hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** *parts from each question.*

1. (a) Prove that

$$\vec{\nabla} \times (\vec{F} \times \vec{G}) = (\vec{\nabla}\cdot\vec{G})\vec{F} - (\vec{\nabla}\cdot\vec{F})\vec{G} + (\vec{G}\cdot\vec{\nabla})\vec{F} - (\vec{F}\cdot\vec{\nabla})\vec{G} \qquad (4\frac{1}{2})$$

(b) If $\vec{F}(t)$ is derivable function of scaler t, prove that the necessary and sufficient condition for $\vec{F}(t)$ to have constant magnitude is $\vec{F}, \frac{d\vec{F}}{dt} = 0$. (4½)

(c) Find the equation for the plane pendicular to the vector $\vec{A} = 2\vec{i} + 3\vec{j} + 6\vec{k}$ and passing through the terminal point of the vector $\vec{B} = \vec{i} + 5\vec{j} + 3\vec{k}$. (4½)

P.T.O.

2. (a) Solve : $(x + 2y - 1)dx = (x + 2y + 1)dy$ (4½)

(b) Solve : $4y^2p^2 + 2pxy(3x + 1) + 3x^3 = 0$ (4½)

(c) Solve : $x(1 - x^2)y^2\frac{dy}{dx} + (2x^2 - 1)y^3 = ax^3$ (4½)

3. (a) Solve : $(D^2 - 1)_y = e^{-x}\sin e^{-x} + \cos e^{-x}$ (5)

(b) Solve : $(D^2 + a^2)_y = \sec ax$ (5)

(c) Prove that the Wronskian of any two solutions of the linear second order homogeneous differential equation

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$$

where $a_0(x) \neq 0$ and $a_0(x)$, $a_1(x)$ and $a_2(x)$ are continuous $\forall\ x \in (a, b)$ are linearly dependent iff their Wronskian is identically zero. (5)

4. (a) Solve :

$$\cos^2x\, y'' - 2\sin x \cos x\, y' + \cos^2x\, y = 0$$ (4½)

(b) Solve :

$$xy'' - (2x + 1)y' + (x + 1)y = (x^2 + x - 1)e^{2x}$$ (4½)

(c) Solve : $(x^3 - x)y'' + y' + n^2x^3y = 0$ (4½)

5. (a) Solve : $(x + 2)y'' - (2x + 5)y' + 2y = (x + 1)e^x$ by the method of variation of parameters. (4½)

(b) Solve an initial value problem

$$y'' - 2y' - 3y = 2e^x - 10 \sin x$$

$$y(0) = 2$$

$$y'(0) = 4$$

by the method of undetermined coefficients. (4½)

(c) Solve :

$$x^2y'' - xy' + y = \log x - (\log x)^2 \qquad (4½)$$

6. (a) Solve the following simultaneous differential equations :

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t \qquad (4½)$$

(b) Solve :

$$\frac{dx}{1} = \frac{dy}{2} = \frac{dz}{5z + \tan(y - 2x)} \qquad (4½)$$

(c) Solve :

$$zy(1 + 4xz)dx - xz(1 + 2xz)dy - xy\,dz = 0 \qquad (4½)$$