This question paper contains 5 printed pages.

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Your Roll No. .....

B.Sc. / II **JS**

MATHEMATICAL SCIENCES

OPERATIONAL RESEARCH

Paper IV— Optimization – II

Time : 3 hours *Maximum Marks* : 55

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any five questions.
Marks are indicated against each question.

1. (*a*) Is it correct to say that in quadratic programming problem the objective function and the constraints both should be quadratic? If not, give your comments. 4

   (*b*) Derive the Kuhn-Tucker necessary conditions for an optimal solution to a quadratic programming problem. 7

2. (*a*) Define the following terms:—

   (*i*) Network

   (*ii*) Path and Chain

   (*iii*) Cut

   (*iv*) Static flow in a network. 4

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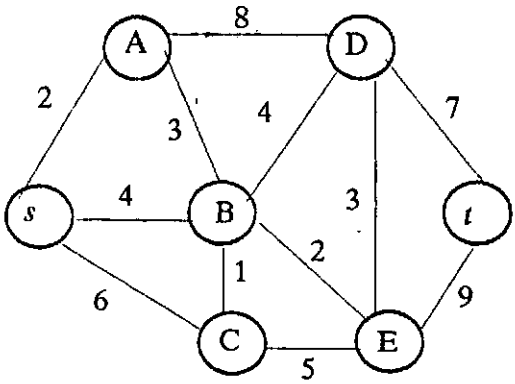
(*b*) Find the shortest path (by complete enumeration method) from s to t in the following network. Numbers on arcs represent actual distances between the corresponding pair of nodes.



3. A small project is composed of seven activities whose time estimates are listed in the following table:

Estimated Duration (Weeks)

Activity	*Optimistic*	*Most likely*	*Pessimistic*
1–2	1	1	7
1–3	1	4	7
1–4	2	2	8
2–5	1	1	1
3–5	2	5	14
4–6	2	5	8
5–6	3	6	15

(*i*) Find the expected duration and variance of each activity.

(*ii*) Calculate the expected project completion time and its variance.

(*iii*) Determine the probability that the project will be completed in 19 weeks and what due date has 90% chance of being met. 11

4. The table below provides costs and time estimates of seven activities of a project.

Activity (I–J)	*Time Estimates* (Weeks)		*Direct Cost Estimates* (Rs.'000)	
	Normal	*Crash*	*Normal*	*Crash*
1–2	2	1	10	15
1–3	8	5	15	21
2–4	4	3	20	24
3–4	1	1	7	7
3–5	2	1	8	15
4–6	5	3	10	16
5–6	6	2	12	36

(*i*) Determine the critical path and the normal duration and normal cost of the project.

(*ii*) Crash the activities so that the project completion time reduces to 9 weeks. 11

5. Define job-shop and flow-shop problem.

P. T. O.

Solve the following 2-jobs 4 machines job-shop problem. The technological ordering of machines for job 1 is ABCD and for job 2 is DBAC. The processing times of two jobs on the four machines are: 7

	A	*B*	*C*	*D*
Job 1	2	4	5	1
Job 2	2	5	3	6

11

6. (*a*) Explain the two person zero-sum game. Give a suitable example. 4

(*b*) Solve the game whose payoff matrix is given below:

Player A	*Player B*			
	B_1	B_2	B_3	B_4
A_1	3	2	4	0
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

7

7. (*a*) What is the dynamic recursive relation? Describe the general process of backward recursion. 4

(*b*) Use dynamic programming to show that:

$$\sum_{i=1}^{n} P_i \log P_i$$

subject to the constraint

$$\sum_{i=1}^{n} P_i = 1$$

and $P_i \geq 0$ for all i

is minimum when $P_1 = P_2 = \ldots\ldots = P_n = 1/n$. 7

8. (*a*) Define the following terms:

(*i*) Earliest starting time

(*ii*) Latest starting time. 4

(*b*) A salesman has to visit five cities, A, B, C, D and E. The distances (in 100 km) between five cities are given as follows:

		To City				
		A	*B*	*C*	*D*	*E*
From City	*A*	–	17	16	18	14
	B	17	–	18	15	16
	C	16	18	–	19	17
	D	18	15	19	–	18
	E	14	16	17	18	–

Determine the optimal tour. 7

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