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Your Roll No.

109

B.Sc. (G)/II

C

Mathematics—Paper III

(Geometry)

Time : 3 Hours

Maximum Marks : 55

(Write your Roll No on the top immediately on receipt of this question paper.)

Attempt *All* questions,

selecting *two* parts from each question.

1. (a) Show that two circles :

$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0 \text{ and}$$

$$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \text{ are}$$

orthogonal if

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2.$$

P.T.O.

- (b) The Radical Axis of coaxial system of circles is the line :

$$x + y = 0.$$

if one of the members is the unit circle with centre at the origin, then find its limiting points or points of intersection; whichever are real.

- (c) Find the equation of circle which is member of a coaxial system determined by the circle :

$$x^2 + y^2 + 2x = 0 \text{ and}$$

$$x^2 + y^2 - 2x - 4y + 1 = 0$$

and passing through the point (1, -1)

9

2. (a) If normal at a point $(at^2, 2at)$ of the parabola $y^2 = 4ax$ meets it again at $(at'^2, 2at')$ prove that :

$$t' = -\left(t + \frac{2}{t}\right).$$

- (b) Prove that the locus of the middle points of a system of parallel chords of a hyperbola is a straight line.
- (c) Find locus of poles of the tangents to the circle :

$$x^2 + y^2 = 4a^2$$

with respect to the parabola $y^2 = 4ax$. 9

3. (a) Find the locus of a point from which two perpendicular tangents can be drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$: By what name is this curve known ?

- (b) Find the locus of midpoints of chords of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ which are tangent to the circle :}$$

$$x^2 + y^2 = r^2.$$

- (c) Tangent at the point P(9, 4) on the hyperbola

$$\frac{x^2}{45} - \frac{y^2}{20} = 1 \text{ meets its asymptotes at points Q and R.}$$

Find coordinates of Q and R. 9

4. (a) Obtain the equation of the sphere circumscribing the tetrahedron whose faces are :

$$x = 0, y = 0, z = 0$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

- (b) Find the equation of the sphere containing circle

$$x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0;$$

$$x + 2y + 3z - 8 = 0.$$

- (c) Find the limiting points of the coaxial system of spheres determined by the spheres :

$$x^2 + y^2 + z^2 + 3y - 3z + 6 = 0 \text{ and}$$

$$x^2 + y^2 + z^2 + 2x + 4y - 2z + 6 = 0.$$

5. (a) Find the equation of the elliptic cone with vertex at (4, 3, 5) which passes through the circle :

$$x^2 + y^2 + z^2 = 4,$$

$$x + y = 0.$$

- (b) Find the points of intersection of the line :

$$\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2}$$

with the cone

$$11x^2 - 5y^2 + z^2 = 0.$$

- (c) Find the equation of the cylinder whose generators intersect the curve :

$$x^2 + y^2 + z^2 = 9;$$

$$x + y + z - 2 = 0$$

and are parallel to the line :

9

$$\frac{x}{2} = \frac{y}{-1} = \frac{z-1}{-2}$$

6. Trace any *one* of the following conics giving essential details : 10

(i) $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$

(ii) $25x^2 - 120xy + 144y^2 - 2x - 29y - 1 = 0.$