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Your Roll No. ....

110

B.Sc. (G)/II **C**

MATHEMATICS—Paper IV

(Vector Calculus and Differential Equations)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) If $\vec{F}$ and $\vec{G}$ are vector functions, then prove that :

$$div(\vec{F} \times \vec{G}) = (curl\,\vec{F}).\ \vec{G} - (curl\,\vec{G}).\ \vec{F}.$$

(*b*) Find the direction in which the directional derivative of :

$$\phi = x^2 + y^2 - z^2$$

is maximum at the point (1, 3, 2). Find the magnitude of the maximum.

(c) If

$$\phi = 2xz^4 - x^2 y.$$

find $\nabla\phi$ and $|\nabla\phi|$ at the point (2, −2, −1). 5,5,5

2. (a) Solve :

$$xdy - ydy = \sqrt{x^2 - y^2}\ dx.$$

(b) Solve :

$$x = y + a \log p.$$

(c) Solve :

$$y = 2px + y^{n-1} p^n.$$

4½,4½,4½

3. (a) Solve :

$$(D^2 + 4)\, y = \sec^2 x.$$

(b) Solve :

$$(D^2 - 4D + 4)\, y = x^2 + e^x + \cos 2x.$$

(c) Prove that the two solutions $y_1(x)$ and $y_2(x)$ of the linear second order homogeneous differential equation :

$$a_0(x)\, y'' + a_1(x)\, y' + a_2(x)\, y = 0,$$

where $a_0(x) \neq 0$ and $a_0(x), a_1(x), a_2(x)$ are continuous $\forall\, x \in (a, b)$ are linearly dependent iff their Wronskian is identically zero. 4½,4½,4½

4. (a) Solve :

$$xy'' - y' + (1 - x)\, y = x^2 e^{-x}.$$

(b) Solve :

$$\frac{d}{dx}(\cos^2 x . y') + \cos^2 x . y = 0.$$

(c) Solve : 4½,4½,4½

$$x^6 y'' + 3x^5 y' + a^2 y = \frac{1}{x^2}.$$

5. (a) Solve :

$$\frac{d^2 y}{dx^2} + a^2 y = \sec ax$$

by the method of variation of parameters.

(b) Solve :

$$y^{(2)} + 2y^{(1)} + y = x - e^x.$$

by the method of undetermined coefficients.

(c) Solve :

$$(3x + 2)^2 y^{(2)} + 3(3x + 2) y^{(1)} - 36y = 3x^2 + 4x + 1.$$

4½,4½,4½

6. (a) Solve the following simultaneous differential equations :

$$\frac{dx}{dt} + 7x + y = 0$$

$$\frac{dy}{dt} + 2x + 5y = 0.$$

(b) Solve :

$$\frac{-dx}{x(x + y)} = \frac{dy}{y(x + y)} = \frac{dz}{(x - y)(2x + 2y + z)}.$$

(c) Solve : 4½,4½,4½

$$z(1 - z^2)\, dx + z\,dy - (x + y + xz^2)\, dz = 0.$$