[This question paper contains 4 printed pages.]

**4688** *Your Roll No.* .....

**B.Sc. (G)/II/(NS)** **AS**

MATHEMATICAL SCIENCES (Operational Research)

Paper IV – Optimization – II

*Time : 3 Hours* *Maximum Marks : 55*

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*Answer any* **Five** *questions.*
**All** *questions carry equal marks.*

1. What is the general form of a Quadratic Programming Problem.

   Solve the following QPP.

   Max $Z = 2x_1 + 3x_2 - 2x_2^2$

   Subject to $x_1 + 4x_2 \leq 4$

   $x_1 + x_2 \leq 2$

   $x_1, x_2 \geq 0$ (2+9)

2. (a) A construction company is preparing a PERT chart for the arts museum. The set of activities, their predecessors requirements and their three time estimates are given in the table given below :

P.T.O.

| Activity | Predeccesors | Time Estimates | | |
|---|---|---|---|---|
| | | Optimistic | Pessimistic | Most Likely |
| A | – | 2 | 4 | 3 |
| B | – | 8 | 8 | 8 |
| C | A | 7 | 11 | 9 |
| D | B | 6 | 6 | 6 |
| E | C | 9 | 11 | 10 |
| F | C | 10 | 18 | 14 |
| G | C, D | 11 | 11 | 11 |
| H | F, G | 6 | 14 | 10 |
| I | E | 4 | 6 | 5 |
| J | I | 3 | 5 | 4 |
| K | H | 1 | 1 | 1 |
| L | K | 1 | 1 | 1 |

(i) Draw the PERT Chart.

(ii) Determine the critical path.

(iii) The contract specifies a penalty of Rs. 5000 per week, if the completion of the project extends beyond 37 weeks. What is the probability that the construction company will have to pay a maximum penalty of Rs. 15,000 ? (9)

(b) Differentiate between PERT and CPM. (2)

3. State Bellman's Principle of optimality. Use Dynamic Programming Approach to divide a positive quantity c into n parts such that their product is maximum. (2+9)

4. (a) Discuss briefly various types of problems studied under theory of sequencing. (3)

   (b) Give an algorithm for solving a 2-job 4-machine job shop problem where Technological ordering of jobs on the machines is specified as follows :

| | | | | |
|---|---|---|---|---|
| Job 1 | A | B | C | D |
| Job 2 | D | B | A | C |

(8)

5. (a) Solve the following game between two players $P_1$ & $P_2$, where the payoff matrix is :

$$\begin{array}{c} \\ P_1 \end{array} \begin{array}{c} \begin{array}{cc} P_2 & \end{array} \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{array}. \qquad (5)$$

   (b) Show the equivalence of a two person zero sum game to a LPP. (6)

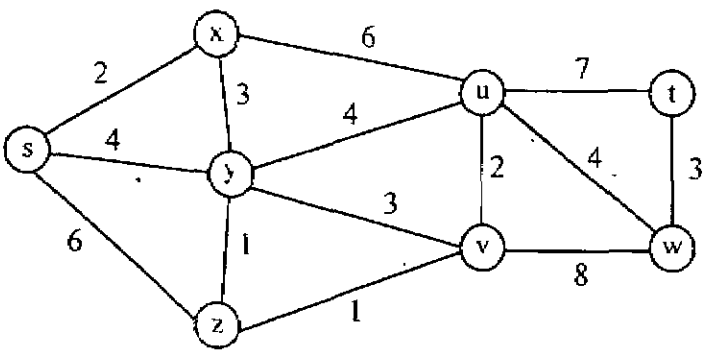
6. (a) Give Node – arc concept of flow in the network. (6)

   (b) Using Labelling Technique, determine the Maximal flow in the network whose capacitance matrix is given as follows :

| | s | x | y | z | u | t |
|---|---|---|---|---|---|---|
| s | – | 10 | 12 | 8 | 0 | 0 |
| x | 0 | – | 6 | 14 | 5 | 0 |
| y | 0 | 0 | – | 12 | 9 | 6 |
| z | 0 | 0 | 1 | – | 3 | 5 |
| u | 0 | 0 | 0 | 1 | – | 7 |
| t | 0 | 0 | 0 | 0 | 0 | – |

Also make diagram of the network. (5)

7. (a) Using complete enumeration method, find shortest path from s to t in the following network. (Nos. along the arcs represent actual distances between the corresponding pair of nodes) (6)




(b) Derive Kuhn-Tucker conditions for solving a general QPP. (5)