[This question paper contains 4 printed pages.]

8043 *Your Roll No.* ....

B.Sc. (G)/II **JS**

MATHEMATICAL SCIENCES (STATISTICS)

Paper III – Statistical Methods – II

Time : 3 hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **six** *questions.*
Marks are indicated against each question.

1. (a) Explain the terms

   (i) Null hypothesis

   (ii) Type I and type II errors

   (iii) Critical region

   (b) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same against that they are not, at 5% level of significance. (3,4)

P.T.O.

2. (a) If $F(n_1, n_2)$ represents an F-variate with n_1, n_2 d.f., prove that $F(n_1, n_2)$ is distributed $1/F(n_2, n_1)$ variate. Also deduce that

$$P(Fn_1, n_2 \geq C) = P(Fn_2, n_1 \leq \tfrac{1}{C})$$

(b) Show that for a 2×2 contingency table where the frequencies are $\begin{array}{c|c} a & b \\ \hline c & d \end{array}$,

$$\chi^2 = \frac{(a+b+c+d)(ad-bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$ (3,3)

3. (a) Obtain the unbiased estimate for population mean and variance. Also obtain the standard error for the mean when sample size is large.

(b) If $X_1, X_2, ---, X_n$ is a random sample from a normal population with mean μ and variance σ^2, prove that ns^2/σ^2 has a Chi-square distribution with $n - 1$ d.f., where $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$. (3,4)

4. (a) Define chi-square variate and obtain its sampling distribution.

(b) Eleven school boys were given a test in statistics. They were given a month's tuition and a second test was held at the end of it. Do the marks give evidence that the students have benefited by the extra coaching ?

Boys	1	2	3	4	5	6	7	8	9	10	11
Marks in I test	23	20	19	21	18	20	18	17	23	16	19
Marks in II test	24	19	22	18	20	22	20	20	23	20	18

(Given that $t_{10}(.05) = 2.23$) (3,3)

5. (a) Show that the mode of the F-distribution with n_1, n_2 d.f. is given by $\frac{n_2(n_1-2)}{n_1(n_2+2)}$ and is always less than unity.

   (b) Show that, for large d.f., t-distribution tends to standard normal distribution. (3,3)

6. (a) Define Fisher's z-transformation. How will you use it for testing the hypothesis $\rho = \rho_0$?

   (b) Let $X_1, X_2, ---, X_n$ be a random sample from a population with continuous density. Show that $Y_1 = \min(X_1, X_2, ---, X_n)$ has exponential distribution with parameter $n\lambda$ if and only if each X_i is exponential with parameter λ. (3,4)

7. (a) A die is thrown 60 times with the following results :

Face	1	2	3	4	5	6
Frequency	8	7	12	8	14	11

Discuss whether the dice may be regarded as an unbiased one, assuming that $\rho(\chi^2 > 11.07) = .05$ with 5 d.f.

(b) Define Student's t and Fisher's t statistics. Show that Student's t-statistic can be regarded as a particular case of Fisher's-t statistic. (3,3)

8. (a) If X_1 and X_2 are independent chi-square variates with n_1 and n_2 d.f. respectively, then show that

$$U = \frac{X_1}{X_1 + X_2} \text{ and } V = X_1 + X_2$$ are independently distributed.

(b) Two random samples of sizes 8 and 11, drawn from normal populations, are as follows :

Sample	Size	Sum of observations	Sum of squares of observations
1	8	9.6	61.52
2	11	16.5	73.26

Is the difference between the variances significant ? (Given that $F_{7,10}(.05) = 3.15$) (4,3)