This question paper contains 4+2 printed pages]

Your Roll No................

227

B.Sc. Prog./II **C**

MA-202—MATHEMATICS-II-Algebra

& Differential Equations

(For Physical Sciences/Applied Physical Sciences)

(Admissions of 2008 and onwards)

Time : 3 Hours *Maximum Marks* : 112

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

Unit I

1. (*a*) Define a subgroup of a group G. Prove that a non-empty finite subset H of a group G is a subgroup of G iff H is closed under the operation of G. 7½

(*b*) Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$, for all a and b in G. 7½

(*c*) Is U(8) a cylic group ? Justify your answer. 7½

P.T.O.

2. (*a*) Let G be a group and $a, b \in G$. If $ab = ba$ and g.c.d.$[0(a), 0(b)] = 1$, show that $0(ab) = 0(a) \cdot 0(b)$. 7½

(*b*) Let G be a group and $a \in G$. Define : 7½

$$N(a) = \{x \in G \mid xa = ax\},$$

Show that $N(a)$ is subgroup of G. Find :

$$N\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) \text{ in } G = GL\ (2, R),$$

the group of 2×2 non-singular matrices over Reals R.

(*c*) Let : 7½

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{pmatrix},$$

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}.$$

Compute $\beta^{-1} \alpha\beta$ and find its order.

3. (*a*) Define normal subgroup of a group G. Let

$$H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \middle| \, a, b, d \in R, ad \neq 0 \right\}.$$ 7½

Is H a normal subgroup of GL(2, R) ?

(*b*) Let H be a subgroup of G. Show that H is normal subgroup of G, if and only if $xy \in H$ implies $yx \in H, \forall \, x, y \in G$. 7½

(*c*) If G is a finite group, show that $\frac{G}{H}$ is also finite for any normal subgroup H of G. Is the converse true ? Justify your answer. 7½

Unit II

4. (*a*) Solve : 6,5½

(*i*) $(4xy^2 + 6y)\, dx + (5x^2y + 8x)dy = 0$

(*ii*) $y = 2xp + y^2p^3, \; p = \frac{dy}{dx}$.

(*b*) Solve by the method of variation of parameters : 11½

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}.$$

(*c*) Prove that there exists two linearly independent solutions $y_1(x)$ and $y_2(x)$ of the second order homogeneous linear differential equation : 11½

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0,$$

where a_0, a_1, a_2 are continuous real valued functions of x defined on (a, b) and $a_0(x) \neq 0$ for any x in (a, b), such that its every solution $y(x)$ may be written as :

$$y(x) = c_1y_1(x) + c_2y_2(x), x \in (a, b),$$

where c_1 and c_2 are suitably chosen constants.

5. (*a*) Solve : 11

$$\frac{dx}{dt} + \frac{dy}{dt} - y = 2t + 1$$

$$2\frac{dx}{dt} + 2\frac{dy}{dt} + y = t.$$

(*b*) Solve : 11

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0.$$

(*c*) An 8-lb weight is attached to the lower end of a coil spring suspended from a fixed support. The weight comes to rest in its equilibrium position, thereby stretching the spring 6 in. The weight is then pulled down 9 in, below its equilibrium position and released at $t = 0$. The medium offers a resistance in pounds numerically equal to $4\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second. Determine the displacement of the weight as a function of the time. 11

6. (a) Find the general integral of : 11

$$z(xp - yq) = y^2 - x^2,$$

where

$$p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$

(b) Find the complete integral of : 11

$$p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2),$$

where

$$p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$

(c) Reduce the equation : 11

$$\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form.