[This question paper contains 6 printed pages.]

234 *Your Roll No.* .....

**B.Sc. (Prog.) / II** **C**

OPERATIONAL RESEARCH

Paper OR-201 : Optimization

(Admissions of 2005 and onwards)

*Time :* 3 *hours* *Maximum Marks : 112*

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*Attempt any* **five** *questions.*

1. (a) What is Operational Research ? What are the limitations of Operational Research ? Discuss the applications of Operational Research in decision making. (10)

(b) A company manufactures two kinds of machines, each requiring a different manufacturing technique. The deluxe machine requires 18 hours of labour, 8 hours of testing and yields a profit of Rs. 400/-. The standard machine requires 3 hours of labour, 4 hours of testing and yields a profit of Rs. 200/-. There are 800 hours of labour and 600 hours of testing available each month. A marketing forecast

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has shown that the monthly demand for the standard machine is to be more than 150. The management wants to know the number of each model to be produced monthly that would maximize total profit. Formulate a linear programming problem and solve it graphically. (12½)

2. (a) Define a convex set. Show that the set

$$S = \left\{ (x_1, x_2) ; 3x_1^2 + 2x_2^2 \leq 6 \right\}$$

is convex. (11)

(b) If S and T are any two convex sets in $R^n$, then show that for all scalars $\alpha$, $\beta$, the set $\alpha S + \beta T$ is also a convex set. (11)

3. (a) Illustrate graphically the following special cases of linear programming problem :

(i) Multiple optimal solutions

(ii) No feasible solution and

(iii) Unbounded problem (12)

(b) Find all basic feasible solutions for the system

$x_1 - 4x_2 + x_3 = 8$

$x_1 + 2x_2 + x_4 = 4$

$x_1, x_2, x_3, x_4 \geq 0$ (10)

4. (a) Explain the two phase method for solving a given linear programming problem. (8½)

(b) Use simplex method to solve the following L.P.P. :

Maximize $Z = 4x_1 + 10x_2$

Subject to the constraints :

$$2x_1 + x_2 \leq 50$$

$$2x_1 + 5x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 90$$

and $x_1, x_2 \geq 0$ (14)

5 (a) What is duality ? Prove that the dual of the dual of a given primal is again primal. Give the economic interpretation of dual variables. (12)

(b) Give the dual of the following L.P.P. :

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to the constraint :

$$2x_1 + 3x_2 - 5x_3 \geq 2$$

$$3x_1 + x_2 + 7x_3 = 3$$

$$x_1 + 4x_2 + 6x_3 \leq 5$$

$x_1, x_2 \geq 0$ and $x_3$ is unrestricted (10)

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6. (a) The optimal simplex table of the linear programming problem

Max $Z = 3x_1 + 5x_2$

Subject to the constraints :

$$3x_1 + 2x_2 \leq 18$$

$$x_1 + 2x_2 \leq 4$$

$$x_2 \leq 6$$

and $x_1, x_2 \geq 0$

is given as follows :

| | | $C_j$ | 3 | 5 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $C_B$ | $X_B$ | b | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $S_3$ |
| 3 | $x_1$ | 2 | 1 | 0 | 1/3 | 0 | −2/3 |
| 0 | $S_2$ | 0 | 0 | 0 | −2/3 | 1 | 4/3 |
| 5 | $x_2$ | 6 | 0 | 1 | 0 | 0 | 1 |
| Z=36 | $Z_j$-$C_j$ | | 0 | 0 | 1 | 0 | 3 |

(i) Suppose variable $x_6$ is added to the given LP problem. Then obtain an optimal solution to the resulting LP problem. It is given that the coefficients of $x_6$ in the constraint of the problem are 1, 1 and 1, and that its coefficient in the objective function is 2.

(ii) Discuss the effect on the optimal basic feasible solution by adding new constraints $x_1 + x_2 \leq 10$ and $2x_1 + x_2 \leq 8$. (14½)

(b) What is integer linear programming ? Explain the merits and demerits of "rounding off" a continuous optimal solution to an LPP to obtain an integer solution. (8)

7. (a) Consider the problem of assigning five operators to five machines with the following assignment cost matrix

| | | Machines | | | | |
|---|---|---|---|---|---|---|
| | | I | II | III | IV | V |
| | A | 10 | 5 | 13 | 15 | 16 |
| Operators | B | 3 | 9 | 18 | 3 | 6 |
| | C | 10 | 7 | 2 | 2 | 2 |
| | D | 5 | 11 | 9 | 7 | 12 |
| | E | 7 | 9 | 10 | 4 | 12 |

What are the operator-machine pair that shall minimize the cost ? (10)

(b) A company has four warehouses a, b, c and d. It is required to deliver a product from these warehouses to three customers A, B and C. The

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warehouses have the following amounts in stock.

| Warehouse | : | a | b | c | d |
|---|---|---|---|---|---|
| No. of units | : | 15 | 16 | 12 | 13 |

and the customers requirements are

| Customer | : | A | B | C |
|---|---|---|---|---|
| No. of units | : | 18 | 20 | 18 |

The table below shows the costs of transporting one unit from warehouse to the customer.

| | | Warehouse | | | |
|---|---|---|---|---|---|
| | | a | b | c | d |
| Customer | A | 8 | 9 | 6 | 3 |
| | B | 6 | 11 | 5 | 10 |
| | C | 3 | 8 | 7 | 9 |

Find the optimal transportation routes. (12½)